The Geometry of the Power Rule

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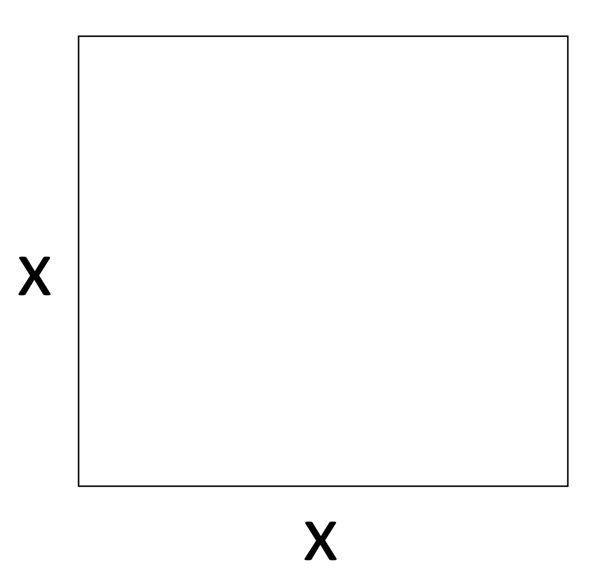
Introduction

- There is a direct relationship between the power rule and the geometry of cubes
- The change in the area of an n-dimensional cube correlates with the derivative of the function x^n

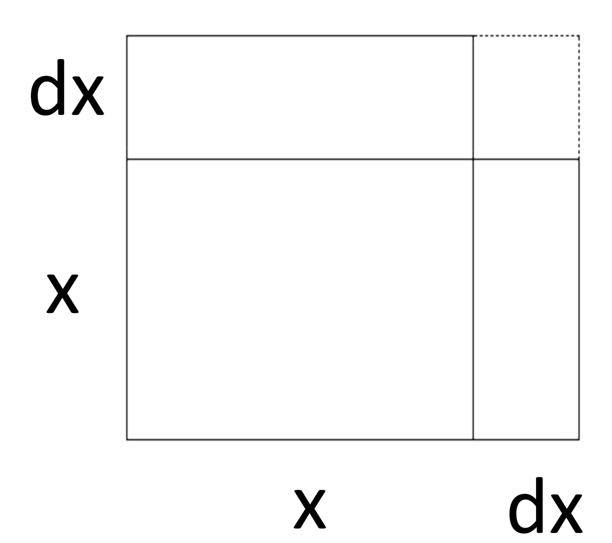
Example 1

Here is a square with side length x



$A = x^2$

Here is the same square with a small length dx added to two of the sides



$A^* = x^2 + 2x\,dx + dx^2$

The change in area is equal to the new area, A^* minus the old area, A

$A^* - A = x^2 + 2x\,dx + dx^2 - x^2$

$= 2x\,dx + dx^2$

Because dx is infinitesimally small, dx^2 can be treated as zero, leaving us with just 2x dx

Notice that this is the same as the derivative of x^2, 2x dx

Equation 1:

$$E_{m,n} = 2^{n-m}\binom{n}{m}$$

This equation tells us how many m-dimensional objects make up an n-dimensional analogue of a cube

Function	Coefficient of 1st derivative	#of sides (from equation)	Factor
	2	4	2
	3	6	2
	4	8	2
	n	2n	2

dx to half of the sides, therefore multiplying by 2 will account for all the sides

Function	Coefficient	Equation	Factor
	1	1	1
	4	8	2
	12	24	2
24x	24	32	1.333

Function	Coefficient	Equation	Factor
	1	1	1
	8	16	2
	56	112	2
	336	672	1.333

Although these factors are seemingly unrelated, there is actually a pattern that emerges

Factor	Ratio of factors
1	0.5
2	1.0
2	1.5
1.333	2.0
0.666	2.5

Conclusion

- The derivative of a function is closely related to the geometry of an n-dimensional cube
- The pattern demonstrated above relates the number of m-dimensional objects making up an n-dimensional cube with the coefficients of derivatives

References

Advisor: Isaac DeFrain

Inspired by "Derivative formulas through geometry | Chapter 3, Essence of calculus" by 3Blue1Brown https://www.youtube.com/watch?v=S0_qX4VJhMQ