Purcell's Scallop Theory

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ABSTRACT

Purcell's Scallop Theorem is the movement of objects through fluids of various viscosities based on different motions. In our experiment, laundry detergent and water will be the viscous fluids used. Detergent represents the viscous fluid and water is the standard. An object with periodic motion will move forward in water and no net motion forward in more viscous fluid. In periodic motion, the forward and reverse movements cancel each other out in the detergent due to a lack of inertia. An object performing a corkscrew motion will move freely in any fluid. By using periodic and corkscrew motion mechanisms in water and detergent, the result of the theory will be observed. Mathematical calculations demonstrate how the periodic motion cancels and the corkscrew motion does not. This theory duplicates natural bacteria and the adjustment of their flagella to navigate forward in viscous fluids.

INTRODUCTION

This phenomenon is observed as bacteria travels through the bloodstream of organisms as well as in water. Observing the way these bacteria move allow advancements to be made in medication distribution as well as furthering studies of bacterial diseases.

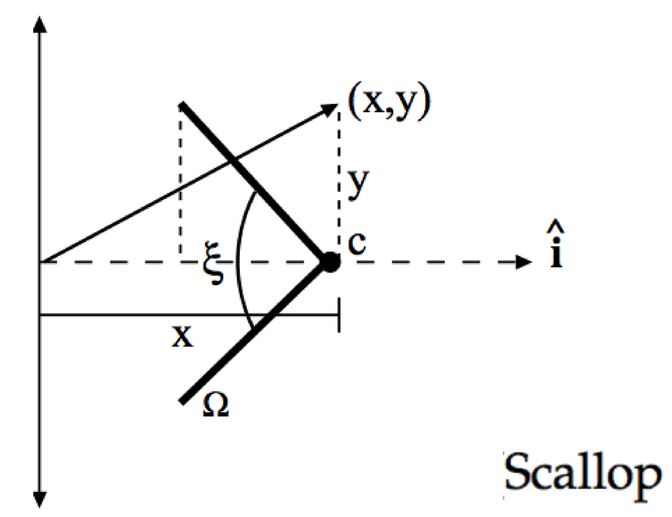
OBJECTIVES

Prove that a propeller with a periodic reciprocal motion will not move in a fluid with a higher viscosity while a propeller with a corkscrew motion will.Also to view how the Reynold's number and size of an object effects the objects ability to move in liquid.

METHODS

- Proved why periodic motion cancels itself with calculations.
- Tested theory with trials of toy fish in fluids with varying viscosities.
- The Reynold's number of an organism swimming determines what kind of liquids it can swim through based on size.
- Ignore size, the motion of the organism dictates what fluids it can swim through.

RESULTS



Scallop

(Scallop Theorem) *Consider a swimmer whose motion is described by*

$$\dot{c}(t) = V(\xi(t))\dot{\xi}(t), \quad t \in \mathbb{R}$$

with $V \in \mathcal{L}^1(\mathcal{S})$. *Then for every T-periodic (reciprocal) stroke* $\xi \in W^{1,\infty}(\mathbb{R};\mathcal{S})$, *one has*

$$\Delta c = \int_0^T \dot{c}(t)dt = 0$$

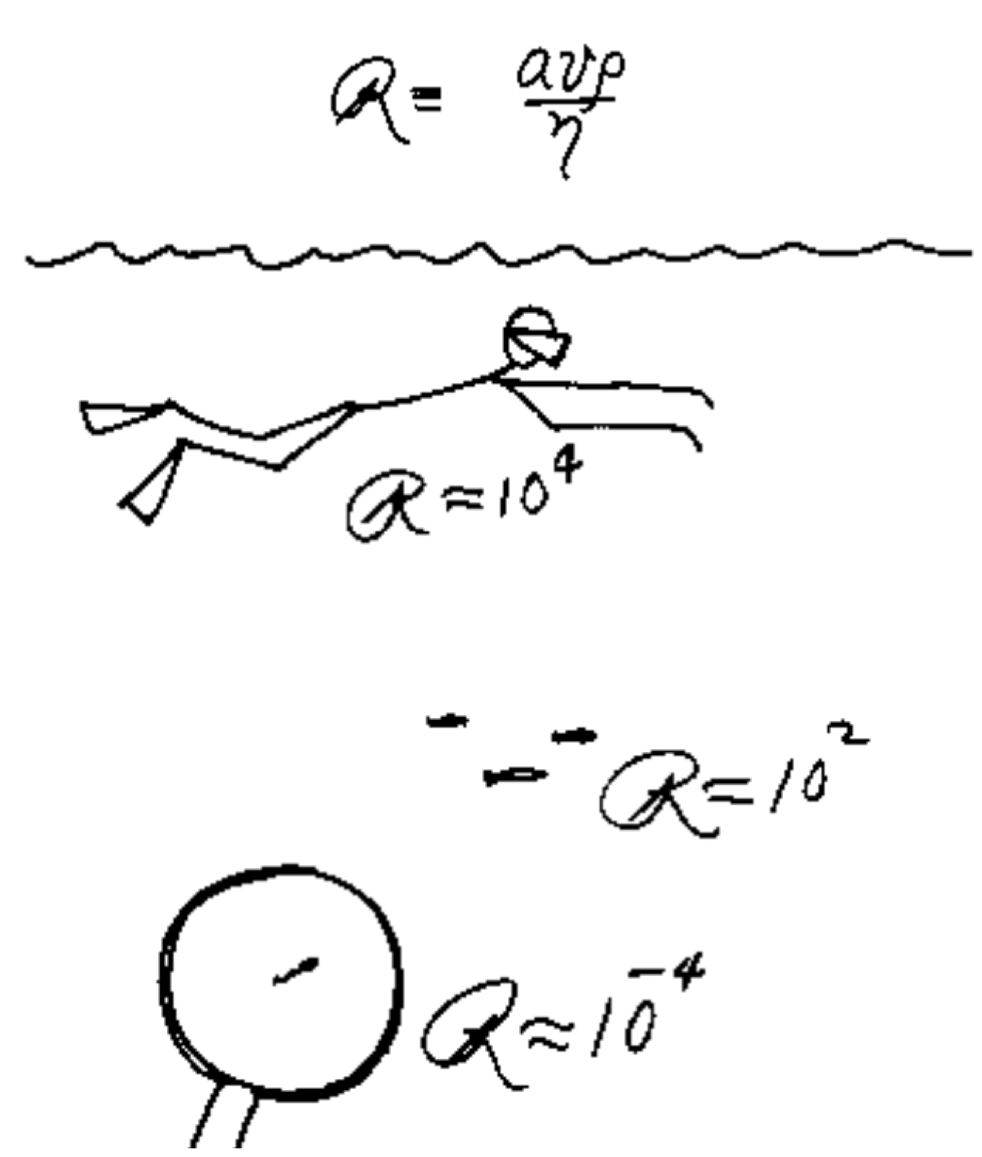
Proof: Define the primitive of V by

$$\Psi(s) = \int_0^s V(\alpha)d\alpha$$

so that by the Fundamental Theorem of Calculus $\Psi'(\xi) = V(\xi)$. Thus

$$\begin{aligned}
\Delta c &= \int_0^T \dot{c}(t)dt, \quad \text{Since distance = rate} \times \text{time} \\
&= \int_0^T V(\xi(t))\,\dot{\xi}(t)dt, \\
&= \int_0^T \Psi'(\xi(t))\,\dot{\xi}(t)dt, \quad \text{By definition of } \Psi(\xi) \\
&= \int_0^T \frac{d}{dt}\Psi(\xi(t))\,dt, \quad \text{By Chain Rule} \\
&= \Psi(\xi(T)) - \Psi(\xi(0)) = 0
\end{aligned}$$

Stokes Drag Law $\quad \mathbf{F} = 6\pi\eta R(\xi)\mathbf{u}$




Both fish moving freely in water (low viscosity)


ry fish not being able to move in the fluid with high viscosity
ile Nemo fish can.

)NCLUSIONS

e theory was able to be proven through the physical monstration as well as the math calculations. The culations cancel each other out to prove objects nnot move in high viscosities with periodic motion.

FUTURE WORK

This theory can be used in research to build microscopic robots. These bots would be about the same size as bacteria making it a challenge to get them to swim in human blood. These bytes theoretically can target a specific part of the body known to have cancer and distribute medication and/or fight the cancer. The challenge is to find a way to make the bots swim by understanding how microorganisms swim in fluids.

References

Dr. Shawn Ryan, Scallop Theorem: Discussion and Proof, November 7, 2010 (Unpublished)

E. M. Purcell. Life at low reynolds number. American Journal of Physics, 45:3–11, 1977.

Acknowledgments

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