Experiment 1A:
Speed of Light Measurement
(Foucault Method)

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Objective

• The objective of this experiment was to measure the speed of light using the same method developed by the French Physicist, Jean Bernard Leon Foucault, in 1862.
Theoretical Background

- The speed of light is one of the most important constants of nature.
- After Einstein presented the *Special Theory of Relativity*, it was established that the speed of light is an upper limit to the velocity of any object.
- Throughout history, there have been many attempts to measure the speed of light, such as those by Galileo, Rømer, Fizeau, and Foucault.
Theoretical Background Cont.

- In 1638, Galileo proposed having two covered lanterns at the tops of two hills, located about a mile apart.
- After one lantern is uncovered, the person holding the second lantern uncovers theirs.
- By measuring the time elapsed between the first person uncovering their lantern and when that person sees the second lantern, the speed of light could then be calculated by dividing twice the distance between them by the elapsed time.
- However, because of human reaction times and the greatness of the speed of light, Galileo could only determine that the speed was much greater than could be determined by use of his procedure.
• Rømer was the first to prove light has a finite speed.
• It is known that the moon would be eclipsed for the same amount of time.
• Because he noticed that the eclipse appeared to be longer when Earth was moving away from Jupiter than when it was moving toward Jupiter, he attributed this to the finiteness of the speed of light.
• Observations made between 1671–1673 were the basis for his calculation.
• He calculated the speed of light to be $2.1 \times 10^8$ m/s. (29% error)
In 1829, Fizeau's designed a method consisted of a spinning wheel with slits. Light would travel through a slit, bounce off of a mirror, and either pass back through the same slit or get blocked by the next slit.

Fizeau used specific values for the distance between the mirror and the wheel, the number of slits on the wheel, and the rate of rotation of the wheel to find the speed of light.

The speed of light he found was $3.15 \times 10^8$ m/s (5% error).
Theoretical Background Cont.

- Below is the Diagram for the Foucault Method showing the optical path of the laser in the setup.
Setup
Setup Cont.

• A laser alignment bench and an optics bench are placed end-to-end.
• The laser is placed on the laser alignment bench.
• The rotating mirror, $M_r$, is placed on the end of the optics bench opposite the laser.
• The laser is aimed such that the beam hits the center of $M_r$, by use of alignment jigs.
• Align the mirror such that the reflective side is perpendicular to the beam, thus reflecting the beam back toward the laser.
• Place the first lens, $L_1$, which has a focal length of 48mm so that it is aligned with the 93cm mark on the optics bench. If necessary, move $L_1$ relative to its holder to again center the beam on the mirror.
Setup Cont.

- Place the second lens, \( L_2 \), which has a focal length of 252mm so that it is aligned with the 62.2cm mark. If needed, move the lens relative to the holder to ensure that the beam is centered on \( M_r \).
- Place the measuring microscope on the optics bench so that the right edge of the mounting stage is aligned with the 82cm mark. \( L_2 \) may need to be re-adjusted to center the beam again.
- Place the fixed mirror, \( M_f \), 2-15m from \( M_r \). The angle formed between the axis of the optics bench and a line connecting \( M_r \) and \( M_f \) should be approximately 12°.
- Position \( M_r \) such that the beam is directed toward \( M_f \). Then adjust the position of \( M_f \) so that the beam strikes it approximately in the center.
- Place a piece of paper in front of \( M_f \) and slide \( L_2 \) back and forth along the optics bench to focus the beam to the smallest possible point.
- Adjust the alignment screws on the back of \( M_f \) so the beam is reflected back to the center of \( M_r \).
Setup Cont.

• Place the polarizers between the laser and $L_1$. Start with the polarizers completely crossed, then rotate one until the image in the microscope is bright enough to be visible.

• Bring the cross-hairs of the microscope into focus by sliding the microscope eyepiece up and down.

• Focus the microscope by loosening the lock-screw and sliding the scope up and down. The point image should become visible. Focus until the image becomes as sharp as possible.
Procedure

• Turn the motor of $M_r$ on so that it rotates in the clockwise direction and increase the revolutions per second slowly to 1000. Then, hold the button which causes the motor to reach its maximum rotational speed.

• Measure the corresponding deflection of the image point by adjusting the micrometer knob on the microscope.

• After allowing the motor to come to rest, turn the motor on in the counter-clockwise direction and measure the resulting deflection after reaching the maximum rotational speed.

• This process is then repeated for 5-10 trials for three different distances between $M_r$ and $M_f$. 

Parameters

- Laser used:
  - He-Ne source
  - 633nm wavelength
  - 4mW power
Derivation

Figure 2a: When $M_2$ is at angle $\theta$, the laser beam is reflected to point $S$ on $M_1$.

\[ S_1 - S = D(2\theta_1 - 2\theta) = D[2(\theta + \Delta\theta) - 2\theta] = 2D\Delta\theta \]

\[ \Delta s' = \Delta s = (i/o)\Delta S = \frac{A}{D + B} \Delta S \]

\[ \Delta s' = \frac{2DA}{D + B} \frac{\Delta\theta}{\Delta \omega} \]

\[ \Delta \theta = \frac{2D \omega}{c} \]

\[ \Delta s' = \frac{4AD^2 \omega}{c(D + B)} \]

\[ c = \frac{4AD^2 \omega}{(D + B) \Delta s'} \]
Derivation Cont.

• To analyze our data we used the equation

\[ c = \frac{8\pi AD^2(Rev/sec_{cw} + Rev/sec_{ccw})}{(D + B)(s'_{cw} - s'_{cw})} \]

• Where
  – C = speed of light
  – A = the distance between \( L_2 \) and lens \( L_1 \) minus the focal length of \( L_1 \)
  – B = the distance between lens \( L_2 \) and \( M_r \)
  – D = the distance between the \( M_r \) and \( M_f \)
  – \( s' \) = the location of the dot in the microscope.
### Data

#### Distance One

- \( D := 10\text{m} \)
- \( L_1 := .9295\text{m} \)
- \( L_2 := .6225\text{m} \)
- \( f_1 := .048\text{m} \)
- \( M_{R} := .135\text{m} \)

\[
\begin{align*}
    s_{cw} := & \begin{pmatrix}
        .009145 \\
        .00914 \\
        .009138 \\
        .00913 \\
        .009135
    \end{pmatrix} \\
    m s_{ccw} := & \begin{pmatrix}
        .00850 \\
        .00850 \\
        .00850 \\
        .00850 \\
        .00850
    \end{pmatrix} \\
    m w_{cw} := & \begin{pmatrix}
        1510 \\
        1513 \\
        1510 \\
        1511 \\
        \text{rev} \frac{\text{s}}{\text{s}}
    \end{pmatrix} \\
    w_{ccw} := & \begin{pmatrix}
        1514 \\
        1515 \\
        1514 \\
        1514 \\
        \text{rev} \frac{\text{s}}{\text{s}}
    \end{pmatrix}
\end{align*}
\]

\[
c_{A} := \frac{4A_1 \cdot D^2 \cdot (w_{cw} + w_{ccw})}{(D + B)(s_{cw} - s_{ccw})} = \begin{pmatrix}
    2.91 \times 10^8 \\
    2.937 \times 10^8 \\
    3.028 \times 10^8 \\
    2.944 \times 10^8 \\
    2.981 \times 10^8 \\
    2.981 \times 10^8 \\
    2.97 \times 10^8 \\
    2.969 \times 10^8
\end{pmatrix} \frac{\text{m}}{\text{s}}
\]

\[
c_{\text{mean}A} := \text{mean}(c_A) = 2.965 \times 10^8 \frac{\text{m}}{\text{s}}
\]
Data Cont.

Speed of Light for Distance 1

- Speed of Light (m/s)
  - c
  - c_A

Trials

- x, t_1

Graph shows the speed of light for different distances with trials ranging from 0 to 10.
Data Analysis

\[
c_A := \frac{4A_1 \cdot D^2 \cdot (w_{cw} + w_{ccw})}{(D + B)(s_{cw} - s_{ccw})} = \begin{pmatrix} 
2.91 \times 10^8 \\
2.937 \times 10^8 \\
3.028 \times 10^8 \\
2.944 \times 10^8 \\
2.981 \times 10^8 \\
2.981 \times 10^8 \\
2.97 \times 10^8 \\
2.969 \times 10^8 
\end{pmatrix} \text{ m/s}
\]

\[
c_B := \frac{4A_2 \cdot D_2^2 \cdot (w_{cw2} + w_{ccw2})}{(D_2 + B_2)(s_{cw2} - s_{ccw2})} = \begin{pmatrix} 
3.161 \times 10^8 \\
3.415 \times 10^8 \\
3.418 \times 10^8 \\
3.289 \times 10^8 \\
3.424 \times 10^8 \\
3.29 \times 10^8 \\
3.565 \times 10^8 \\
3.321 \times 10^8 
\end{pmatrix} \text{ m/s}
\]
Data Analysis Cont.

\[ C_C := \frac{4A_3 D_3^2 (w_{cw3} + w_{ccw3})}{(D_3 + B_3)(s_{cw3} - s_{ccw3})} = \begin{pmatrix} 3.09 \times 10^8 \\ 3.143 \times 10^8 \\ 3.072 \times 10^8 \\ 3.118 \times 10^8 \\ 3.145 \times 10^8 \\ 3.1 \times 10^8 \\ 3.151 \times 10^8 \\ 3.115 \times 10^8 \end{pmatrix} \]

m/s
Data Analysis Cont.

Speed of Light for All Three Trials

<table>
<thead>
<tr>
<th>Trials</th>
<th>Speed of Light (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>t_1, t_2, t_3, x</td>
<td></td>
</tr>
</tbody>
</table>

Graph shows the speed of light for all three trials with different markers and line styles for each attempt. The speed values range from 2x10^8 to 3x10^8 m/s.
Data Analysis Cont.

\[ \delta_w := 1 \frac{\text{rev}}{s} \quad \delta_A := .05\text{cm} \quad \delta_B := .05\text{cm} \quad \delta_D := 1\text{cm} \quad \delta_s := .0000025\text{m} \]

\[ \delta_{cA} := \sqrt{\sigma_{A1}^2 + \sigma_{B1}^2 + \sigma_{D1}^2 + \sigma_{wcw1}^2 + \sigma_{wccw1}^2 + \sigma_{scw1}^2 + \sigma_{sccw1}^2} = 1.784 \times 10^6 \frac{\text{m}}{s} \]

\[ \delta_{cB} := \sqrt{\sigma_{A2}^2 + \sigma_{B2}^2 + \sigma_{D2}^2 + \sigma_{wcw2}^2 + \sigma_{wccw2}^2 + \sigma_{scw2}^2 + \sigma_{sccw2}^2} = 4.772 \times 10^6 \frac{\text{m}}{s} \]

\[ \delta_{cC} := \sqrt{\sigma_{A3}^2 + \sigma_{B3}^2 + \sigma_{D3}^2 + \sigma_{wcw3}^2 + \sigma_{wccw3}^2 + \sigma_{scw3}^2 + \sigma_{sccw3}^2} = 1.381 \times 10^6 \frac{\text{m}}{s} \]
Results

D := 10m
\[ c_{A\text{final}} := 3.0 \times 10^8 \frac{m}{s} \pm 1.8 \times 10^6 \frac{m}{s} \]

%error \:=\: \left| \frac{c_{\text{mean}A} - c}{c} \right| = 1.094\%

D := 4.5m
\[ c_{B\text{final}} := 3.4 \times 10^8 \frac{m}{s} \pm 4.8 \times 10^6 \frac{m}{s} \]

%error2 \:=\: \left| \frac{c_{\text{mean}B} - c}{c} \right| = 12.094\%

D := 14.44m
\[ c_{C\text{final}} := 3.1 \times 10^8 \frac{m}{s} \pm 1.4 \times 10^6 \frac{m}{s} \]

%error3 \:=\: \left| \frac{c_{\text{mean}C} - c}{c} \right| = 3.963\%
Error

• The best result (to within 5%) is obtained for a distance of 10-15m. For the second trial, we used a distance much shorter than this, resulting in a proportionate reduction in accuracy.

• In the third trial, we used a distance which was greater than the radius of curvature of $M_f$, causing problems with the focusing of the image point.

• The alignment of all of the components is very sensitive. Thus, because we likely did not have perfect alignment, this resulted in systematic error for each calculation of the speed of light. Furthermore, small changes in the alignment were likely made due to accidental bumps of the equipment during measurements, creating error in the measurements following the misalignment.
Observations

- The fixed mirror, $M_f$, is sensitive to vibrations. Any jolts to the surface on which it rests results in the mirror becoming misaligned.

- It was necessary for our laser setup to be on the table closer to the fume hood. Otherwise, we could not get the angle to be correct. Also, this stopped the laser beam from hitting anything between $M_r$ and $M_f$. 
Conclusions

• The experiment was quite successful. For the shortest distance, we received a 12% error. However, because proportional reduction in accuracy is expected for smaller distances, this is fairly reasonable.

• For the other two distances, we had errors of 1% and 3.9%, which both indicate good measurements.
References

• Lee, Bruce. 1989. "Speed of Light Apparatus". Physics 335 Laboratory Handout. Cleveland State University, OH.
