Graphs on surfaces and knot theory

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Abstract. Regions of a link diagram can be colored in black and white in a checkerboard manner. Putting a vertex in each black region and connecting two vertices by an edge if the corresponding regions share a crossing yields a planar graph. In 1987 Thistlethwaite proved that the Jones polynomial of the link can be obtained by a specialization of the Tutte polynomial of this planar graph. I will review basic properties of the Tutte polynomial and explain a generalization of Thistlethwaite’s theorem to virtual links. In this case graphs will be embedded into a (higher genus, possibly non-oriented) surface. For such graphs the natural duality can be generalized to a duality with respect to a subset of edges. The dual graph might be embedded into a different surface. For graphs on surfaces there is a generalization of the Tutte polynomial called the Bollobas-Riordan polynomial. I will explain a relation between the Bollobas-Riordan polynomials of dual graphs. This relation unifies various Thistlethwaite’s type theorems.

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