Necessary Condition for Transmission Line Congestion

Shubo Zhang, Advisor: Dr. Hongxing Ye

Department of Electrical Engineering and Computer Science Washkewicz

College of Engineering

Cleveland State University

Introduction

Mixed Integer Programming(MIP) is used to solve Security Constraint Unit Commitment (SCUC) problem in power system operation. MIP determines the optimal dispatch and ON/OFF status of individual generators. Due to large number of security constraints, it is difficult to obtain the exact optimal solution in acceptable time. The introduction of Necessary Condition for Line Congestion is able to eliminate the inactive constraints, significantly reduce the size of security constraint set and improve the performance of the MIP problem [1].

Problem Formulation

The objective of MIP problem is to minimize overall system operating cost while individual generator operating within its physical capability and upholding security constraints. C is the generator cost function, p, I, y, z, r represents dispatch, status, start-up action, shutdown action and ramp rate of individual generator respectively. Γ is the network sensitivity factor matrix, F_l is the power flow limits of transmission line.

$$\min_{(y,z,I,p)\in\mathcal{F}} \sum_g C\left(p_{g,t}, I_{g,t}\right) \tag{1}$$

$$\text{s.t} \quad \sum_g p_{g,t} = \sum_m d_{m,t}, \forall t \tag{2}$$

$$-F_l \le \sum_m \Gamma_{l,m}\left(\sum_{g\in\mathcal{G}(m)} p_{g,t} - d_{m,t}\right) \le F_l, \forall l, t \tag{3}$$

$$I_{g,t}p_g^{\min} \le p_{g,t} \le I_{g,t}p_g^{\max}, \forall g, t \tag{4}$$

$$p_{g,t} - p_{g,(t-1)} \le r_g^u\left(1 - y_{g,t}\right) + p_g^{\min} y_{g,t}, \forall g, t \tag{5}$$

$$-p_{g,t} + p_{g,(t-1)} \le r_g^d\left(1 - z_{g,t}\right) + p_g^{\min} z_{g,t}, \forall g, t \tag{6}$$

Minimum ON/Off Time Limit (7)

Necessary Condition for Line Congestion

The maximum power flow on line l is defined as in (8)

$$\max\left\{\sum_m \Gamma_{l,m}p_{m,t}^{\text{inj}} : \sum_m p_{m,t}^{\text{inj}} = 0, \underline{p}_{m,t}^{\text{inj}} \le p_{m,t}^{\text{inj}} \le \overline{p}_{m,t}^{\text{in}} \forall m\right\} \tag{8}$$

$p_{m,t}^{inj}$ represent the power injection on bus m at time t, $\bar{p}_{m,t}^{inj}$ and $\underline{p}_{m,t}^{inj}$ represents the maximum and minimum power injection. Introducing auxiliary variable $p_{m,t}^{+inj}$

$$p_{m,t}^{\text{inj}+} := p_{m,t}^{\text{inj}} - \underline{p}_{m,t}^{\text{inj}} \tag{9}$$

$$\begin{cases} \overline{p}_{m,t}^{\text{inj}} = \sum_{g\in\mathcal{G}(m)}\left(p_{g,t} + \overline{r}_{g,t}(\boldsymbol{x},\boldsymbol{p})\right) - d_{m,t} + u_{m,t}, \forall m, t \\ p_{m,t}^{\text{inj}} = \sum_{g\in\mathcal{G}(m)}\left(p_{g,t} + r_{g,t}(\boldsymbol{x},\boldsymbol{p})\right) - d_{m,t} - u_{m,t}, \forall m, t \end{cases}$$

Reformulation

$$\max\left\{\sum_m \Gamma_{l,m}p_{m,t}^{\text{inj}+} + f_{l,t}^{\text{vt}} : \sum_m p_{m,t}^{\text{inj}+} = p_t^{\text{vt}}, 0 \le p_{m,t}^{\text{inj}+} \le \overline{p}_{m,t}^{\text{inj}+} \forall m\right\} \tag{10}$$

Denote that $p_t^{\text{vt}} = -\sum_m p_{m,t}^{\text{inj}}$ $\qquad f_{l,t}^{\text{vt}} = \sum_{m=1}^{N_d} \Gamma_{l,m}p_{m,t}^{\text{inj}}$

If there exists an integer $j \in [1, N_d]$, so that (11), (12) and (13) holds, then the flow constraint for line ι (with capacity of F_l) in the positive direction at t is inactive. m_n is is the bus with the n^th largest sensitivity factor Γ_{l,m_n} for line l.

$$\begin{cases} \sum_{n=1}^{j-1}\overline{p}_{m_n,t}^{\text{inj}} \le p_t^{\text{vt}} \le \sum_{n=1}^{j}\overline{p}_{m_n}^{\text{inj}+} & (11) \\ \sum_{n=1}^{j-1}\left(\Gamma_{l,m_n} - \Gamma_{l,m_j}\right)\overline{p}_{m_n,t}^{\text{inj}+} + \Gamma_{l,m_j}p_t^{\text{vt}} + f_{l,t}^{\text{vt}} \le F_l & (12) \\ \Gamma_{l,m_1} \ge \Gamma_{l,m_2} \ge \cdots \ge \Gamma_{l,m_N} & (13) \end{cases}$$

If (10) is feasible, there must exist an integer j satisfying (11). If the shift factors are ordered as (13), the optimal solution to (10) is

$$p_{m_n,t}^{\text{inj}+} = \begin{cases} \overline{p}_{m_n,t}^{\text{inj}+}, & n = 1,\cdots,j-1 & (14) \\ p_t^{\text{vt}} - \sum_{n=1}^{j-1}\overline{p}_{m_n,t}^{\text{inj}+}, & n = j & (15) \\ 0, & n = j+1,\cdots,N_d & (16) \end{cases}$$

And the optimal value is

$$\begin{aligned} &\sum_{n=1}^{j-1}\Gamma_{l,m_n}\overline{p}_{m_n,t}^{\text{inj}+} + \Gamma_{l,m_j}\left(p_t^{\text{vt}} - \sum_{n=1}^{j-1}\overline{p}_{m_n,t}^{\text{inj}+}\right) + f_{l,t}^{\text{vt}} \\ &= \sum_{n=1}^{j-1}\left(\Gamma_{l,m_n} - \Gamma_{l,m_j}\right)\overline{p}_{m_n,t}^{\text{inj}+} + \Gamma_{l,m_j}p_t^{\text{vt}} + f_{l,t}^{\text{vt}} \end{aligned} \tag{17}$$

Simulation

Properly formulate net power injection is the key to determine line congestion. The following experiment is conducted on IEEE 118 bus test system. The bounds of network injection derived by introducing load uncertainty, ϵ_m, and adjustable generation, ΔP_i, limited by ramping constraints, $\bar{r}_i$ and $\underline{r}_i$ [2].

$$\sum_i\left(P_i + \Delta P_i\right) = \sum_m\left(d_m + \epsilon_m\right) \tag{18}$$

$$P_m^{\text{inj}} = \sum_{i\in\mathcal{G}(m)}\left(P_i + \Delta P_i\right) - d_m - \epsilon_m, \forall m \tag{19}$$

$$\underline{r}_i \le \Delta P_i \le \overline{r}_i, \forall i \tag{20}$$

$$\underline{u}_m \le \epsilon_m \le \overline{u}_m \tag{21}$$

$$0 \le P_m^{+\text{inj}} \le \overline{P}_m^{+\text{inj}} = \sum_{i\in\mathcal{G}(m)}\left(\overline{r}_i - r_i\right) - u_m + \overline{u}_m, \forall m \tag{22}$$

Simulation is performed with 6000 MW peak load uncertainty interval bounds are set between 5% to 30% of bus loads on PC Intel i7 3.6GHz. There are 168 *2 security constraints per hour.

Conclusion

In general, over 90% line constraints will not be binding in the MIP problem thus can be eliminated. With the increasing load and uncertainty more lines may be congested. The necessary conditions for line congestion serves as a pre-screening process that reduces security constraints, which drastically decrease MIP solution time and improve solution quality.

Line Congestion Detection Over 24 Hours

Hour	u = 5%d	u = 10%d	u = 20%d	u = 30%d
1	10	14	22	25
2	10	14	22	24
3	8	11	21	24
4	5	9	18	22
5	7	11	19	24
6	14	18	23	27
7	16	18	23	29
8	17	18	28	34
9	14	19	28	47
10	13	16	31	44
11	13	15	31	43
12	13	16	31	44
13	14	18	27	45
14	14	18	27	45
15	15	18	27	42
16	17	17	27	39
17	15	18	27	42
18	14	18	27	45
19	14	17	29	45
20	13	15	31	44
21	14	17	29	45
22	15	18	27	42
23	17	17	24	29
24	17	20	22	27
%Binding	7.91	9.67	15.40	21.75

Future Work

- Determine the dominating constraints within the result by necessary condition of line congestion screening.
- Tightening network power injection interval with statistical Method that determine Unit Commitment information with confident interval.
- Determine the inference of network power injection caused by virtual power plants by Locational Marginal Price forecasting with statistical methods.

Reference

[1] H. Ye and Z. Li, "Necessary Conditions of Line Congestions in Uncertainty Accommodation," *IEEE Transactions on Power Systems*, vol. 31, no. 5, pp. 4165–4166, Sep. 2016.

[2]H. Ye, J. Wang, and Z. Li, "MIP Reformulation for Max-Min Problems in Two-Stage Robust SCUC," *IEEE Transactions on Power Systems*, vol. 32, no. 2, pp. 1237–1247, Mar. 2017.

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