A simple model of eating decisions and weight with rational and forward-looking agents

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Abstract

Several empirical papers in the economics of obesity literature find that changes in aggregate food prices over time have little effect on the population body-mass index or obesity prevalence, while changes in the price of selected food items drastically affects what people eat. We propose a simple dynamic model with rational agents to further examine the impact of changes in food prices and household real income on eating decisions and weight of men and women between 1971 and 2006. We also introduce a new measure for food prices which considers the price per calorie consumed rather than prices of specific food items. After careful calibration of the model using evidence from medical research on obesity, we find that prices determine the allocation of calories across food types, while income determine the total number of calories consumed and thus individuals’ weight. Based on our results, we share the view that taxes on food will impact what people eat but will have limited effect on reducing the population body-mass index or the obesity prevalence.

JEL Classification: I10, D91

Keywords: Obesity, body weight, food prices.

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1 Introduction

Two decades of intense research in the field of economics of obesity have improved our understanding of the impact of food prices on weight and food choices and two critical results have gained wide acceptance. First, several empirical studies show that changes in aggregate food prices over time have little effect on the population body-mass index or obesity prevalence (e.g., Chou et al., 2004, Gelbach et al., 2007). Second, experimental studies show that changes in the price of selected food items drastically affects what people eat. For example, French, Jeffery, Story, Breitlow, Baxter, Hannan, and Snyder (2001) find that a fifty percent price reduction on low-fat snacks in vending machines at schools and work places increase the percentage of low-fat snack sales by ninety three percent.

These two results about the (lack of) effect of food prices on weight and food choices are important because of their influence on the public health debate about the effectiveness of fiscal policies for winning the fight against the obesity epidemic. A common view held by policymakers is that taxes applied selectively to different food items work effectively to reduce consumption of a particular type of food or ingredient (e.g., ban of trans-fats in New-York City) but are unlikely to produce significant changes in body-mass index or obesity prevalence (Powell and Chaloupka, 2009 and Chouinard et al., 2007).

In this paper, we further examine the impact of changes over time in food prices and household real income on individuals’ food choices and weight using a calibrated dynamic model. Our first objective is to contribute to the debate about the impact of food prices and household real income on weight and food choices using a different modeling strategy. Using our calibrated model, we ask how much of the increase in calories consumed away from home as well as changes in weight for men and women in different age groups between 1971 and 2006 can be accounted for by changes in food prices and household real income. A second and perhaps even more critical objective is to use economic theory and available evidence from medical research on obesity to look inside the black box of how people make eating decisions, in particular improve our understanding of what determines the (low) food price elasticity of weight.

Our environment for studying eating decisions and weight accumulation rests upon the following assumptions. First, we use economic theory to model agents’ food choices,
including the total number of calories consumed every day as well as the fraction of calories consumed at home versus away from home. We posit that agents are fully rational and forward-looking. Given household real disposable income and the relative price of a calorie (at home versus away from home), agents decide how much to eat at each location as well as how much of the non-food good to consume to maximize a well-defined objective function. Second, we use medical research on nutrition to link daily calorie intake to weight gain. We assume that weight is a stock variable and calorie intake a flow variable. A weight law of motion relates weight in future periods to weight and food decisions in the current period. Agents gain (lose) weight when total calorie consumption in the current period is greater (less) than the number of calories required to maintain current weight that depends on the agent’s age and is greater for men compared to women. Finally, we use medical research on obesity-related diseases to link agent’s weight to her longevity. We assume that a positive probability exists in each period that agents die and that this probability depends on the agent’s body-mass index. We consider the case where the survival probability is an inverted U-shape function of body-mass index, implying that agents who are either over- or underweight have a greater chance to die. As a result, agents arbitrate the following simple trade-off to smooth utility over time. Increasing food consumption in the current period yields instantaneous utility but also leads to higher weight and increased mortality risks in the future.

We concentrate our analysis on the following facts about weight and calories consumed away from home to evaluate the impact of food prices and income within our model. First, the average weight of adult men and women increased by twenty-two and twenty-three pounds, respectively between 1971 and 2006. Second, the fraction of total daily calories consumed away from home increased from thirty to forty percent for adult men and from eighteen to thirty-five percent for adult women over the same period of time. Finally, we document changes in average weight and calories away from home for men and women in different age groups between 1971 and 2006. Note that gender and age play two key roles in our analysis of weight and food choices reflecting economic and physiological differences between men and women and over the life-cycle. One the physiological side, men can afford to eat more calories every day compared to women without gaining weight.
The relationship between calorie intake and weight also differs along the life-cycle. As individuals age, the number of calories needed to maintain a constant weight declines (see a technical report (2002) published by the Food and Nutrition Board of the Institute of Medicine of the National Academies). On the economic side, household real income fluctuates over the life-cycle influencing food choices and thus weight.

We introduce a new measure for food prices which considers the price per calorie consumed rather than prices of specific food items. Using data on food expenditures as well as calories consumed from the US Department of Agriculture between 1971 and 2006, we calculate price per calorie for food consumed away from home and food consumed at home as the dollar amount spent by households on each food category divided by the number of calories consumed at each location. We find that the relative price per calorie (away from home versus at home) declined by seventeen percent, while household real income increased by twenty-four percent over the same period of time. We use our newly constructed time series for price per calorie as an input into our dynamic model.

Results for the impact of food prices and household income on individuals’ food choices and weight are only interesting and credible if we can justify the choice of the model parameters unequivocally. We use available evidence from medical research on nutrition to calibrate the weight law of motion and medical research on obesity-related diseases to fix the survival probability function. We choose the remaining preferences parameters to match the mean weight and fraction of calories away from home observed in NHANES 1971-75, allowing some preference heterogeneity between men and women. Using our calibrated model, we find that changes in food prices and real income affect eating decisions and weight differently. In a nutshell, prices determine the allocation of calories across food types, while income determine the total number of calories consumed. Taken altogether, however, changes in per calorie food prices and real income account for a large share of the increase in weight and the fraction of calories consumed away from home for men and women in different age groups between 1971 and 2006.

Our results corroborate the existing scientific knowledge about food choices and obesity. Using a fully specified calibrated dynamic model (as opposed to econometrics or field experiments) and time series (as opposed to cross-section) data, we show that changes
in food prices over time account for almost none of the weight gain by Americans men and women in the last thirty years and for more than fifty percent of the increase in calories consumed away from home by men and women. As such, we support the view that taxes on food will impact what people eat but will have limited effect on reducing the population body-mass index or the obesity prevalence.\textsuperscript{1}

The remainder of the paper is organized as follows. In Section 2, we describe data about weight, calories consumed away from home, per calorie food prices and household real income between 1971 and 2006. In Section 3 and Section 4, we develop and calibrate our dynamic model and we conduct our simulations in Section 5. Finally, we offer concluding remarks in Section 6.

\section{Nutritional and Economic Data}

In this section, we use two distinct sample data from the National Health and Nutritional Examination Survey (NHANES) to document changes over time in body-mass index, weight, and the fraction of calories consumed away from home for men and women for the period between 1971 and 2006 (see Table 1).\textsuperscript{2} Data about weight comes from the examination component of NHANES and is measured by trained medical personnel. The fraction of calories consumed away from home, on the other hand, is self-reported by individuals.\textsuperscript{3}

\textsuperscript{1}Note that distortionary taxes on food will result in sub-optimal allocations when agents are fully rational. On the other hand, food taxes will potentially improve welfare when agents have time-inconsistent preferences reflecting commitment issues toward food.

\textsuperscript{2}Body-mass index is a measure of body fat based on height and weight that equally applies to adult men and women. It is calculated as 703 times weight measured in pounds divided by height squared measured in inches squared. Individuals with BMI lower than 18.5 are considered underweight, between 18.5 and 24.9 normal weight, between 25 and 30 overweight, and 30 or greater obese. BMI combined with other information about waist circumference and other factors such as physical activity, cigarette smoking, or low-density cholesterol level gives a risk assessment of developing obese-associated diseases such as heart attacks, diabetes II, strokes, etc.

\textsuperscript{3}The question about where do people eat their meal has changed over time. In NHANES I, individuals can choose among the following four locations: at home, in school, in restaurants, and other, while in NHANES 2005-06, the location question is: “Did you eat this food at home?” and the possible answers
Table 1: Changes in body-mass index, weight, and fraction of calories consumed away from home for men and women

<table>
<thead>
<tr>
<th></th>
<th>1971-75a</th>
<th>2005-06a</th>
<th>%Change</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Men:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BMI</td>
<td>25.9</td>
<td>29.0</td>
<td>12.0</td>
</tr>
<tr>
<td>Weight (lbs.)</td>
<td>175.7</td>
<td>198.2</td>
<td>12.8</td>
</tr>
<tr>
<td>Fraction of Calories Away from Home</td>
<td>29.9</td>
<td>40.5</td>
<td>10.6</td>
</tr>
<tr>
<td><strong>Women:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BMI</td>
<td>25.2</td>
<td>28.8</td>
<td>14.3</td>
</tr>
<tr>
<td>Weight (lbs.)</td>
<td>145.8</td>
<td>168.9</td>
<td>15.8</td>
</tr>
<tr>
<td>Fraction of Calories Away from Home</td>
<td>18.5</td>
<td>35.9</td>
<td>17.4</td>
</tr>
</tbody>
</table>

a Data for the period 1971-75 come from NHANES I, while data for the period 2005-06 come from NHANES.

b \( BMI = 703 \times \frac{Weight}{Height^2} \).

In the last thirty years, men have gained on average 23 pounds and their body-mass index increased from 25.9 (slightly overweight) to 29.0 (borderline obese).\(^4\) The increase in average weight and body-mass index is even more pronounced in percentage terms for women who also gained 23 pounds.\(^5\) In addition, men and women changed their eating

...
habits dramatically and ate out more. The fraction of calories away from home increased by 11 and 17 percentage points for men and women, respectively.

Our goal in this paper is to determine how much of the increase in weight and the fraction of calories consumed away from home can be accounted for by the decline in the relative price of food (at home versus away from home) as well as increases in household real income. To measure changes in food prices, we introduce a new measure which considers the price per calorie consumed rather than prices of specific food items. We calculate price per calorie for food consumed away from home, $p_{A,t}$, and food consumed at home, $p_{H,t}$, as the dollar amount spent by households on each food category divided by the number of calories consumed:

$$p_{A,t} = \frac{\alpha_{A,t} I_t}{\text{Calories}_{A,t}}, \quad p_{H,t} = \frac{\alpha_{H,t} I_t}{\text{Calories}_{H,t}}$$

We calculate the dollar amount spent by households on food as household real disposable income, $I_t$, multiplied by the expenditure share on food consumed away from home $\alpha_{A,t}$ or at home $\alpha_{H,t}$. Information about the expenditure share on food away from home and food at home is obtained from household expenditures data published by the US Department of Agriculture (USDA). The expenditure for food away from home is equal to 3.5 and 4.1 percent, respectively, for the periods 1971-75 and 2005-06. The expenditure for food away from home is equal to 9.9 and 5.7 percent, respectively, for the same time periods. Information about nominal disposable income comes from the Bureau of Economic Analysis (BEA) and we use the consumer price index (CPI) published by BLS to calculate the real disposable income expressed in 2006 dollars. Between 1971 and 2006, household real disposable income increased by 24 percent, while the per calorie price of food consumed away from home declined by seventeen percent (see Table 2).

The price per calorie has two significant advantages over traditional food prices. First, it provides a simple method for aggregating food items in different categories. Second, it is intuitively appealing as it controls for changes in portion sizes at restaurants (Young and Nestle, 2002). Note that the relative price of food away from home is always greater

the interview date in advance, they might change their dietary habits and adjust their calories consumed downward for one day in order to exhibit “good behavior”. As a result, we do not use self-reported data on calorie intake in our analysis.
Table 2: Changes in per calorie food prices and real income

<table>
<thead>
<tr>
<th></th>
<th>1971-1975</th>
<th>2005-2006</th>
<th>% Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative price (Away/Home)</td>
<td>1.61</td>
<td>1.34</td>
<td>-16.7</td>
</tr>
<tr>
<td>Mean Real Income in 2006 $</td>
<td>$59,742</td>
<td>$74,089</td>
<td>24.0</td>
</tr>
</tbody>
</table>

that one, implying that eating out is more costly than eating at home, even after adjusting for differences in the number of calories. In addition, we looked at changes in food prices published by the Bureau of Labor Statistics. We find that the price of food consumed away from home increased by 40 percent from 1971 to 2006. The increase in food prices away from home is clearly at odds with the observed increase in the fraction of calories eaten away from home.

Next, we look at changes over time in the body-mass index and the fraction of calories consumed away from home for men and women in different age groups. Both economic and physiological considerations suggest that age matter for eating decisions. First, income typically increases over the life-cycle reaching its peak around age 50 and then decline at retirement age. If one thinks of calorie consumption as a normal good, variation in income over the life-cycle imply that the number of calorie demanded should increase before age 50 and then decline. Second, the relationship between calorie intake and weight also differs along the life-cycle. As individuals age, the number of calories needed to maintain a constant weight declines (see a technical report (2002) published by the Food and Nutrition Board of the Institute of Medicine of the National Academies).

In Figures 1 and 2, we compare the body-mass index of men and women in different age groups between 1971 and 2006. First, note that body-mass index increases over the life-cycle for both men and women and for both time periods. Second, the body-mass index for men and women increased for all age groups between 1971 and 2006. For men, the largest increase in body-mass index occurs for the age groups 35-44 and 55-64, while for women the largest increase occurs for the age groups 25-34 and 45-54.

In Figures 3 and 4, we compare the fraction of calories consumed away from home by men and women in different age groups between 1971 and 2006. First, the fraction of calories consumed away from home declines with age for both men and women. Second,
Figure 1: Men’s body-mass index by age

Figure 2: Women’s body-mass index by age
the fraction of calories consumed away from home is greater for men compared to women for all age groups. Third, the increase in calories consumed away from home between 1971 and 2006 is somewhat uniform for all age group and for men and women.

Finally, we compare household real disposable income for different age group between 1971 and 2006 in Figure 5. First, real income in 2006 is higher than real income in 1971 for all age group. The increase in real income was the largest for the age group 45-54 and the smallest for young adults in age group 25-34. Second, household income follows a clear pattern over the life-cycle. It increases until age 55 and then declines sharply.

In the next section, we propose a calibrated dynamic model of eating decisions to assess the quantitative impact of changes in food prices and real income on individual weight and their decision to eat out.
Figure 4: Fraction of calories consumed away from home by women by age

Figure 5: Household real income over time by age
3 A Dynamic Optimization Model of Eating Decisions and Weight

Time is discrete and infinite, \( t = 1, 2, \ldots \). In each period, agents decide how much and where to eat (out or at home) as well as non-food consumption. We let \( a_t \) and \( h_t \) be the number of calories consumed away and at home, respectively, and \( c_t^{af} \) represents non-food consumption. Calories away and at home are aggregated using a constant elasticity of substitution (CES) function to obtain food consumption:

\[
c_t^f = (\eta a_t^\rho + (1 - \eta) h_t^\rho)^{\frac{1}{\rho}}
\]

with \( \eta \in (0, 1) \) and \( \rho \in (-\infty, 1] \). Food away and at home are perfect substitutes, Cobb-Douglas, or perfect complements when the parameter \( \rho \) is equal to one, zero, or minus infinity, respectively. The parameter \( \eta \) reflects consumer’s preference for eating at home or eating out, a smaller \( \eta \) indicating that consumers prefer to eat at home.

Preferences of the representative agent are of constant relative risk aversion (CRRA) form and are given by:

\[
U(c_t^f, c_t^{nf}) = \frac{[(c_t^f)^\alpha (c_t^{nf})^{1-\alpha}]^{1-\sigma}}{1-\sigma}
\]

with \((\sigma, \alpha) \in (0, 1) \times (0, 1)\). The inter-temporal elasticity of substitution between consumption in any two periods is equal to \( \frac{1}{\sigma} \) and the smaller \( \sigma \) is, the more willing are households to substitute consumption over time.

Agents make consumption decisions in an environment where their longevity is uncertain. We denote by \( \pi(W_t) \) the conditional probability that a consumer with weight \( W_t \) makes it to the next time period provided that she is alive at the beginning of the period. We assume that the death probability is an inverted U-shape function of body-mass index which implies that agents who are either over- or underweight have a greater chance to die. Finally, death is an absorbing state and consumer receives utility \( U \leq 0 \) forever when they die.

The expected utility discounted to period one is equal to:

\[
\sum_{t=1}^{+\infty} \beta^{t-1} \Pi_{s=1}^{t-1} \pi(W_s)(\pi(W_t)U(c_t^f, c_t^{nf}) + (1 - \pi(W_t))U) = 0
\]
where the parameter $\beta \in (0, 1)$ is the pure time discount factor. Note that it is never optimal for people to eat so much that they would die with certainty since $U(c^f_t, c^n_f)$ is positive and $U \leq 0$.

The inter-temporal weight law of motion links weight in the next period to current weight and calorie consumption:

$$W_{t+1} = W_t + \lambda(a_t + h_t - c(W_t))$$ (5)

where $\lambda > 0$ is a parameter that converts calorie consumption into weight gain and the function $c(W_t) = \mu + \nu W_t$ denotes the daily calorie requirement in order to maintain a constant weight with $\nu > 0$.

Finally, the budget constraint of the representative agent is given by:

$$c^n_f + p_{ht}h_t + p_{at}a_t = I_t$$ (6)

where we normalized the price of non-food to one, $p_{ht}$ and $p_{at}$ are the real price of food at and away from home, respectively, and agents are endowed with real income $I_t$. Note that a more realistic setting would include credit markets to provide an extra channel that agents could use to smooth utility over time. The rise in credit availability and use between 1971 and 2006 might be an important factor for explaining individuals’ decisions to eat at restaurants (Levy, 2007). In our model, agents smooth utility over time through the following simple trade-off. Increasing food consumption in the current period yields instantaneous utility but also leads to higher weight and increased mortality risks in the future.

For any given sequence of prices and income, $\{p_{ht}, p_{at}, I_t\}_{t \geq 1}$, and an initial weight, $W_1$, the representative agent chooses an optimal sequence of calories from food away and at home as well as non-food consumption, $\{a_t, h_t, c^n_f\}_{t \geq 1}$, to maximize the expected utility discounted to period one in equation (4) subject to the budget constraint (6), the weight law of motion (5), the food aggregation equation (2), and non-negativity constraints for calorie and non-food consumption.

We substitute the weight law of motion into the objective function in equation (4). The consumption of food away from home, $a_t$, and food at home, $h_t$, appear as follows in
the objective function:

\[
... + \beta^{t-1} \pi(W_1) \times ... \times \pi(W_{t-1}) \times \\
\left( \pi(W_t) \frac{[(\eta a^0 + (1 - \eta)h_t^0)^\rho (I_t - p_{ht}h_t - p_{at}a_t)^{1-a}]}{1 - \sigma} + (1 - \pi(W_t)) \nu \right) + \\
\beta^t \pi(W_1) \times ... \times \pi(W_t) \left( \pi(W_t + \lambda(a_t + h_t - c(W_t))) \frac{[(c_{t+1}^f)^{a} (c_{t+1}^{nf})^{1-a}]}{1 - \sigma} \right) + \\
(1 - \pi(W_t + \lambda(a_t + h_t - c(W_t))) \nu) + ...
\]

We take first-order conditions with respect with food away from home, \(a_t\), and food at home, \(h_t\). In the Appendix, we show that the system of first-order conditions is given by:

\[
\frac{\alpha \eta a_t^{\rho-1}}{\eta a_t^\rho + (1 - \eta)h_t^\rho} - \frac{p_{at}(1 - \alpha)}{I - p_{at}a_t - p_{ht}h_t} = -\frac{\lambda \beta}{1 - \sigma} \pi'(W_{t+1}) \frac{U(c_{t+1}^f, c_{t+1}^{nf}) - \bar{U}}{U(c_t^f, c_t^{nf})}
\]

\[
\frac{\alpha(1 - \eta)h_t^{\rho-1}}{\eta a_t^\rho + (1 - \eta)h_t^\rho} - \frac{p_{ht}(1 - \alpha)}{I - p_{at}a_t - p_{ht}h_t} = -\frac{\lambda \beta}{1 - \sigma} \pi'(W_{t+1}) \frac{U(c_{t+1}^f, c_{t+1}^{nf}) - \bar{U}}{U(c_t^f, c_t^{nf})}
\]

Note that consumer’s utility might not be strictly concave because the survival probability depends on consumer’s weight. As a result, it is not clear whether first-order conditions are sufficient for optimality. Although we do not offer a formal proof, we check in our computer simulations that the allocations that satisfy the first-order conditions in equation (8) is also utility-maximizing (locally).

We analyze the above system of equations when prices, income, and quantities are constant over time. That is, when \(a_t = a^*, h_t = h^*, W_t = W^*, p_{at} = p_{at}^*, p_{ht} = p_{ht}^*,\) and \(I_t = I^*\). The weight law of motion in equation (5) provides the relationship between weight and calorie intake, \(W^* = \frac{a^* + h^* - \mu}{\alpha}\). After rearranging slightly, the steady-state version of the first-order conditions in equation (8) is given by:

\[
\frac{\eta a^*^{\rho-1} - (1 - \eta)h^*^{\rho-1}}{\eta a^*^{\rho} + (1 - \eta)h^*^{\rho}} = \frac{1 - \alpha}{\alpha} \frac{p_a - p_h}{I^* - p_a a^* - p_h h^*}
\]

\[
\frac{\alpha \eta a^*^{\rho-1}}{\eta a^*^{\rho} + (1 - \eta)h^*^{\rho}} - \frac{p_a(1 - \alpha)}{I^* - p_a a^* - p_h h^*} = -\frac{\lambda \beta}{1 - \sigma} \pi'(a^* + h^* - \frac{\mu}{\nu}) \left(1 - \frac{\bar{U}}{U(c_{t}^{f*}, c_{t}^{nf*})} \right)
\]

The steady state food away from home, \(a^*\), and food at home, \(h^*\), are obtained by solving the above system of equations (9).
In the reminder of the paper, we explain our method to calibrate the three key parts of our model: the weight law of motion, the survival probability function, and the deep preference parameters. We then use the calibrated model to conduct a lab experiment where we assess the impact of food price and real income on weight and the fraction of calories away from home.

4 Calibration

We use medical research on obesity to calibrate the weight law of motion and the survival probability function. We then chose the remaining preference parameters to match the average weight and calories away from home for men and women observed in the NHANES I sample.

4.1 Law of motion

The weight law of motion in equation (5) contains three distinct important parameters. First, the constant $\lambda$ converts excess calorie intake into weight gain. Second, the two parameters $(\mu, \nu)$ determine the calorie requirement to maintain a constant weight with $\mathcal{c}(W_t) = \mu + \nu W_t$.

According to the dietary guidelines from the US Department of Agriculture, people gain ten pounds per year if they eat an extra one hundred calories every day above and beyond the recommended daily calorie intake. As a result, we fix $\lambda = \frac{10}{100 \times 365} = 2.7397 \times 10^{-4}$.

According to the Food and Nutrition Board of the Institute of Medicine of the National Academies (technical report 2002), the minimum number of calories required to maintain a constant weight depends on gender, age, weight, height, and the activity level. For men, this daily calorie requirement is given by:

$$c^m = 662 - 9.53 \times Age + Activity \times (7.23 \times Weight + 13.706 \times Height)$$

(10)

where age is expressed in years, weight in pounds, height in inches, and the activity level is equal to 1, 1.12, 1.27, and 1.45, when the activity level is sedentary, low active, active, and very active, respectively. Note that the number of daily calories required to maintain
constant weight declines as men get older but increases with the activity level, weight, and height. Men’s mean age and height in the NHANES I sample is equal to 41.2 and 69.6 inches, respectively. Assuming a low level of activity (activity=1.12), the calorie requirement equation for men becomes:

\[ c^m(W_t) = 1338.2 + 8.09W_t \]  

(11)

The previous equation identifies the parameters \( \mu^m = 1338.2 \) and \( \nu^m = 8.09 \). For example, a man of age 41.2 who exercise moderately and weighs 170 pounds needs to eat 2715 calories every day to maintain a constant weight.

For women, the amount of calories needed to maintain a constant weight is given by:

\[ c^w = 356 - 6.91 \times Age + Activity \times (4.25 \times Weight + 18.44 \times Height) \]  

(12)

Women’s mean age and height in the NHANES I sample is equal to 40.3 and 63.6 inches, respectively. Assuming a low level of activity (activity=1.12), the calorie requirement equation for women becomes:

\[ c^w(W_t) = 1391.1 + 4.76W_t \]  

(13)

The previous equation identifies the parameters \( \mu^w = 1391.1 \) and \( \nu^w = 4.76 \). For example, a woman of age 40.3 who exercise moderately and weighs 130 pounds needs to eat 2009 calories every day to maintain a constant weight.

In the previous section, we showed that the steady state weight is equal to \( W^* = \frac{a^* + h^* - \mu}{\nu} \). As a result, the steady state weight for men and women is equal to:

\[
W^{*m} = \frac{a^* + h^* - 1338.2}{8.09}, \quad W^{*w} = \frac{a^* + h^* - 1391.1}{4.76}
\]  

(14)

### 4.2 Survival Probability Function

We posit that the survival probability function \( \pi(W_t) \) is given by the following function form:

\[ \pi(W_t) = 1 - \pi_a \times hr(W_t) \]  

(15)

where \( \pi_a \in (0,1) \) captures the death likelihood for reasons unrelated to obesity and \( hr(W_t) \) denotes the increased likelihood of death due to being over or underweight. Throughout
the rest of the paper, we set $\pi_a = 0.01$. We use the work of Allison et al. (1999) to calibrate the hazard rate function $hr(W_t)$. Allison et al. (1999) report the hazard ratios of death based on six large prospective cohort studies where subjects are placed into two distinct groups: the control group is comprised of individuals whose body-mass index (BMI) is between twenty-three and twenty-five; the treated group consists of individuals with BMI higher than twenty-five. In Table 3, we present their results and it is clear that the probability of dying tends to increase with BMI. For example, in the Alameda Country Health Study, people with BMI between thirty and thirty-five have 1.36 more chance to die compared to people with normal weight, and the hazard ratio increases to 2.79 for people with BMI greater than thirty-five.\footnote{Fontaine, Redden, Wang, Westfall, and Allison (2003) calculate years of life lost due to obesity. The maximum years of life lost for white men twenty to thirty years of age with a severe level of obesity (BMI > forty-five) is thirteen years and eight for white women.}

We use the regression algorithm in Judd (1998, p.223) to approximate the hazard rate function with a linear combination of Chebyshev polynomials, $\{T_i\}_{i=0}^n$. The algorithm consists of evaluating a linear combination of Chebyshev polynomials of degree $n$ polynomials at $m > n$ pre-determined nodes on a given weight interval $[w_{\text{min}}, w_{\text{max}}]$. The nodes are equal to $w_k = (z_k + 1)(\frac{w_{\text{max}} - w_{\text{min}}}{2}) + w_{\text{min}}$ for all possible $k = 1, ..., m$ with $z_k = -\cos(\frac{2k-1}{2m}\pi)$. In our case, we chose a polynomial of degree six, seven nodes, and we fix the bounds of the weight interval to correspond to a body-mass index of $b_{\text{min}} = 19$ and $b_{\text{max}} = 37$. Since the seven weight nodes correspond to seven values of the body-mass index, we use the research by Allison to evaluate the survival probability at all seven weight nodes with $y_k = hr(w_k)$.\footnote{There is a one-to-one relationship between body weight and body-mass index since $BMI = 703 \times \frac{\text{Weight}}{\text{Height}^2}$.} Finally, the algorithm provides a formula for the coefficients of the Chebyshev polynomials with $a_i = \sum_{k=1}^{k=7} w_k T_i(z_k) / \sum_{k=1}^{k=7} T_i(z_k)^2$ for all possible $i = 0, ..., 6$. In our case, the estimated coefficients $\{\hat{a}_i\}$ are equal to $\{1.309, 0.302, 0.317, 0.039, 0.064, 0.035, 0.026\}$. As a result, the survival probability function is approximated by the following function:

$$hr(W_t) = \sum_{i=0}^{i=6} a_i T_i(2 \frac{W_t - w_{\text{min}}}{w_{\text{max}} - w_{\text{min}}} - 1)$$ (16)

for any possible weight in the interval $[w_{\text{min}}, w_{\text{max}}]$. In Figure 6, we present the survival probability as a function of the body-mass index. As expected from the Allison’s data,
Table 3: Hazard Ratios for various BMI categories

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Alameda</td>
<td>1.39</td>
<td>1.00</td>
<td>0.98</td>
<td>0.86</td>
<td>1.20</td>
<td>1.26</td>
<td>1.23</td>
<td>1.36</td>
<td>2.79</td>
</tr>
<tr>
<td>Framingham</td>
<td>1.12</td>
<td>1.00</td>
<td>0.96</td>
<td>1.11</td>
<td>1.04</td>
<td>1.08</td>
<td>1.41</td>
<td>1.60</td>
<td>1.94</td>
</tr>
<tr>
<td>Tecumseh</td>
<td>1.20</td>
<td>1.00</td>
<td>1.18</td>
<td>0.89</td>
<td>1.12</td>
<td>0.92</td>
<td>0.94</td>
<td>1.45</td>
<td>1.87</td>
</tr>
<tr>
<td>Cancer Society</td>
<td>1.07</td>
<td>1.00</td>
<td>1.02</td>
<td>1.06</td>
<td>1.08</td>
<td>1.14</td>
<td>1.21</td>
<td>1.35</td>
<td>1.72</td>
</tr>
<tr>
<td>Nurses</td>
<td>1.06</td>
<td>1.00</td>
<td>0.92</td>
<td>0.96</td>
<td>1.09</td>
<td>1.21</td>
<td>1.32</td>
<td>1.49</td>
<td>1.89</td>
</tr>
<tr>
<td>NHANES</td>
<td>1.04</td>
<td>1.00</td>
<td>0.96</td>
<td>1.11</td>
<td>0.96</td>
<td>1.40</td>
<td>1.06</td>
<td>1.33</td>
<td>1.68</td>
</tr>
</tbody>
</table>

Source: Allison, Fontaine, Manson, Stevens, and VanItalie (1999)
the survival probability is U-shape. It reaches it maximum when the body-mass index is equal to .

4.3 Preferences

We are now left with calibrating six preferences parameters, \((\alpha, \eta, \sigma, \rho, \bar{U}, \beta)\). First, since the utility function \(U(c^f_t, c^{nf}_t)\) is positive, we fix \(U = 0\) so that death coming from excess eating is never an optimal choice.

Second, we use the research of Reed et al. (2005) which estimates the elasticity of substitution between food away from home and food consumed at home. They find that both types of foods are substitutes and as a result, we fix \(\rho = 0.75\).

Third, due to lack of available estimates in the economic literature, we arbitrarily fix the inter-temporal elasticity of substitution \(\sigma = 0.5\) and the pure time discount factor \(\beta = 0.96\). Since we did not choose these parameters to match some statistics of the data, we will conduct sensitivity analysis to determine the importance of these two parameters for our results.

Finally, we use the two first-order conditions in equation (9) to determine \((\alpha, \eta)\) to match the observed average weight and fraction of calories consumed away from home for men and women in the NHANES I sample. For the period 1971-75, men’s average
weight and fraction of calorie consumed away from home was equal 175.7 pounds and 29.9 percent, respectively (see Table 1). According to equation (11), the total number of calories to maintain a weight of 175.7 pounds is equal to 2760.9, implying that the steady state value for calories consumed at home and away from home is equal to $a^* = 825.5$ and $h^* = 1935.4$, respectively. Per calorie prices of food away from home and food at home are equal to $p_a^* = 4.23 \times 10^{-3}$ and $p_h^* = 2.97 \times 10^{-3}$, respectively. Using the information about real income from Table 2, the steady state of non-food daily consumption is equal to $c_{nf}^* = 154.4$. As a result, the parameters $(\alpha, \eta)$ are obtained by solving the following system of equations:

\[
\frac{\alpha \eta 825.5^{-0.25}}{\eta 825.5^{0.75} + (1 - \eta) 1935.4^{0.75}} - \frac{4.23 \times 10^{-3} (1 - \alpha)}{154.4} = \frac{\alpha (1 - \eta) 1935.4^{-0.25}}{\eta 825.5^{0.75} + (1 - \eta) 1935.4^{0.75}} - \frac{2.97 \times 10^{-3} (1 - \alpha)}{154.4}
\]

For men, we find that $\eta^m = 0.53$ and $\alpha^m = 0.06$.

Finally, we determine the coefficient $\alpha^w$ and $\eta^w$ to match the observed average weight and fraction of calories consumed away from home for women in the NHANES I sample. For the period 1971-75, women’s average weight and fraction of calorie consumed away from home was equal 145.8 pounds and 18.5 percent, respectively (see Table 1). According to equation (13), the total number of calories to maintain a weight of 145.8 pounds is equal to 2085, implying that the steady state value for calories consumed at home and away from home is equal to $a^* = 385.7$ and $h^* = 1699.3$, respectively. Using the information about food prices and income from Table 2, the steady state consumption of non-food is equal to $c_{nf}^* = 157.0$. As a result, the parameters $(\alpha, \eta)$ are obtained by solving the
following system of equations:

\[
\frac{\alpha \eta^{385.7^{-0.25}}}{\eta^{385.7^{-0.75}} + (1 - \eta)1699.3^{0.75}} - \frac{4.23 \times 10^{-3}(1 - \alpha)}{157.0} = \frac{\alpha(1 - \eta)1699.3^{-0.25}}{\eta^{385.7^{0.75}} + (1 - \eta)1699.3^{0.75}} - \frac{2.97 \times 10^{-3}(1 - \alpha)}{157.0}
\]

\[
\frac{\alpha \eta^{385.7^{-0.25}}}{\eta^{385.7^{0.75}} + (1 - \eta)1699.3^{0.75}} - \frac{4.23 \times 10^{-3}(1 - \alpha)}{157.0} = \frac{-2.7397 \times 10^{-4} \times 0.96}{0.5} \pi'(145.8)
\]

For women, we find that \( \eta^w = 0.49 \) and \( \alpha^w = 0.04 \).

Note that men and women differ considerably in their preferences for food versus non-food goods and food at home versus food away from home. The food share, \( \alpha \), and the preference parameter for food away from home, \( \eta \), are greater for men compared to women. Note that the heterogeneity across gender is not counter-intuitive since men tend to eat more than women and they also eat more away from home. In the next section, we use the calibrated model to assess the impact of changes in relative food prices and real income on eating habits and weight of Americans between 1971 and 2006.

5 Simulations

We perform the following experiments. First, we change the price per calorie of food away from home from its 1971 value, \( p_{a1971} = 4.23 \times 10^{-3} \), to its 2006 value, \( p_{a2006} = 3.70 \times 10^{-3} \) leaving all other parameters of the model constant. From the first-order conditions in equation (9), we calculate the new steady state values for food away from home and food at home. We then calculate the steady-state weight for men and women from equations (11) and (13), respectively, as well as the resulting body-mass index. We report results of the first experiment in the first column of Table 4. For men, the fraction of calories away from home increases by 12 percentage points from 30 percent (calibrated value from Table 1) to 42 percent. For women, the fraction of calories consumed away from home increases.
by 8 percentage points from 19 percent (calibrated value) to 27 percent. The impact on agent’s weight is small. Men and women only gain one pound as their weight increases to 177 and 146 pounds, respectively.

Table 4: Average body-mass index, weight, calorie requirement, and fraction of calories consumed away from home for men and women

<table>
<thead>
<tr>
<th></th>
<th>$p_a$</th>
<th>$p_h$</th>
<th>Income</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Men:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BMI</td>
<td>26</td>
<td>26</td>
<td>28</td>
<td>29</td>
</tr>
<tr>
<td>Weight (lbs.)</td>
<td>177</td>
<td>175</td>
<td>194</td>
<td>197</td>
</tr>
<tr>
<td>Fraction of Calories Away from Home (%)</td>
<td>42</td>
<td>26</td>
<td>34</td>
<td>36</td>
</tr>
<tr>
<td><strong>Women:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BMI</td>
<td>25</td>
<td>25</td>
<td>29</td>
<td>30</td>
</tr>
<tr>
<td>Weight (lbs.)</td>
<td>146</td>
<td>144</td>
<td>166</td>
<td>170</td>
</tr>
<tr>
<td>Fraction of Calories Away from Home</td>
<td>27</td>
<td>14</td>
<td>21</td>
<td>24</td>
</tr>
</tbody>
</table>

The second experiment consists of changing the price per calorie of food at home from its 1971 value, $p_{h1971} = 2.97 \times 10^{-3}$, to its 2006 value, $p_{h2006} = 2.72 \times 10^{-3}$ leaving all other parameters constant. We calculate the new steady state value for food away from home and food at home as well as weight and body-mass index as explained above. We report the result in the second column of Table 4. For men, the fraction of calories away from home decreases by 4 percentage points from 30 percent to 26 percent. For women, the fraction of calories away from home decreases by 5 percentage points from 19 percent to 14 percent. Again, the decline in food prices has little impact on agent’s weight.

The third experiment consists of changing household real disposable income from its 1971 value $I_{1971} = $59,742 to its 2006 value $I_{2006} = $74,089 (see Table 2) leaving all other parameters constant. We report the results in the third column of Table 4. Changes in

---

8For men, results slightly overshoots the data as the observed fraction of calories consumed away from home in 2006 is equal 41 percent. For women, results do not fully account for the observed change in the data as the fraction of calories consumed away from home by women in 2006 is equal to 36 percent.
income account for a large fraction of the observed change in individual’s weight. The steady state weight of men and women increases to 194 and 166 pounds, respectively.\footnote{In the data, the mean weight of men and women in 2006 is equal to 198 and 169 pounds, respectively (see Table 1).}

Changes in income also induce a reallocation effect. As agents become richer, the fraction of calories consumed away from home increase from 30 percent to 36 percent for men and from 19 percent to 24 percent for women.

Finally, the fourth experiment consists of changing food prices and income all at once. For men, the model predicts that a weight equal to 197 pounds and the fraction of calories away from home is equal to 36 percent. For women, the model predicts that a weight equal to 170 pounds and the fraction of calories away from home is equal to 24 percent.

The lessons learned from the model for eating decisions and weight can be summarized as follows. Changes in food prices have an “allocation” effect. As the price of one food category changes, households substitute from one food category to another. Between 1971 and 2006, the decline in the relative price of food (away from home versus at home) account for about half of the increase in the fraction of calories eaten away from home. Changes in food prices, however, have little impact on total calories consumed and weight. Changes in income, on the other hand, have a large impact on weight. Between 1971 and 2006, much of the increase in weight and body-mass index can be accounted for by increase in household real disposable income.\footnote{Dolar (2009) shows that the positive relationship between body-mass index and household income holds for men in several cross-sections of NHANES. For women, however, body-mass index is negatively related to household income suggesting that some other force not captured in our model is at work. We leave the task of reconciling the pattern differences for body-mass index and household income in cross-section and time-series data for men and women for future research.}

Next, we look at the model predictions for men and women in different age groups. We allow the preference parameters $\eta$ and $\alpha$ to differ by age and gender and we re-calibrate the model as follows. First, we denote by $W(g, i)$ the average weight for men and women $g \in \{m, f\}$ in age group $i \in \{25 – 34, 35 – 44, 45 – 54, 55 – 64\}$ observed in NHANES 1971-75. From equation (11) and equation (13), we calculate the total number of calories for men and women in each age group to maintain a constant weight. Second, we determine the number of calories consumed away from home as well as calories consumed at home.
by gender, \( a(g, i) \) and \( h(g, i) \), respectively as total calories times the fraction of calories eaten at each location. Finally, the parameters \((\alpha(g, i), \eta(g, i))\) are obtained by solving the following system of equations:

\[
\frac{\alpha(g, i)\eta(g, i)a(g, i)^{-0.25}}{\eta(g, i)a(g, i)^{0.75} + (1 - \eta(g, i))h(g, i)^{0.75}} = \frac{4.23 \times 10^{-3}(1 - \alpha(g, i))}{I(i) - p_a a(g, i) - p_h h(g, i)}
\]

\[
\frac{\alpha(g, i)(1 - \eta(g, i))h(g, i)^{-0.25}}{\eta(g, i)a(g, i)^{0.75} + (1 - \eta(g, i))h(g, i)^{0.75}} = \frac{2.97 \times 10^{-3}(1 - \alpha(g, i))}{I(i) - p_a a(g, i) - p_h h(g, i)}
\]

\[
\frac{\alpha(g, i)\eta(g, i)a(g, i)^{-0.25}}{\eta(g, i)a(g, i)^{0.75} + (1 - \eta(g, i))h(g, i)^{0.75}} = \frac{4.23 \times 10^{-3}(1 - \alpha(g, i))}{I(i) - p_a a(g, i) - p_h h(g, i)}
\]

\[= -\frac{2.7397 \times 10^{-4} \times 0.96}{0.5} \pi'(W(g, i))\]

where information about household real disposable income at age \( i \), \( I(i) \), is obtained in Figure 5. Calibrated values for \( \eta \) and \( \alpha \) are reported in Table 5.

**Table 5: Preferences Parameters \( \eta \) and \( \alpha \) by age and gender**

<table>
<thead>
<tr>
<th>Age</th>
<th>( \eta )</th>
<th>( \alpha )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Men</td>
<td>Women</td>
</tr>
<tr>
<td>Age 25-34</td>
<td>0.55</td>
<td>0.50</td>
</tr>
<tr>
<td>Age 35-44</td>
<td>0.54</td>
<td>0.49</td>
</tr>
<tr>
<td>Age 45-54</td>
<td>0.51</td>
<td>0.48</td>
</tr>
<tr>
<td>Age 55-64</td>
<td>0.50</td>
<td>0.46</td>
</tr>
</tbody>
</table>

The parameters \( \eta \) and \( \alpha \) differ considerably by age and gender. First, the food away from home share, \( \eta \), is always higher for men compared to women for all age groups and decreases with age for both men and women. Second, the food share parameter, \( \alpha \), is greater for men compared to women and also declines with age for men.

In the fifth experiment, we assess the impact of decline in food prices and increase in real income (altogether) for men and women in the age group 25-34. In the first-order condition in equation (9), household income \( I \) is equal to household real disposable income.
for the age group 25-34 in 2006 value $I_{25-34}^{2006} = 59,609$ (see Figure 5). Per calorie prices for food away from home and food at home are equal to their 2006 values, $p_a^{2006} = 3.70 \times 10^{-3}$ and $p_h^{2006} = 2.72 \times 10^{-3}$, respectively. Finally, the parameters $(\eta, \alpha)$ are equal to their calibrated values in Table 5 for men and women. That is, $\eta(m, 25 – 34) = 0.55$ and $\alpha(m, 25 – 34) = 0.07$ for men and $\eta(m, 25 – 34) = 0.50$ and $\alpha(m, 25 – 34) = 0.05$ for women.

We solve for the steady state value for food away from home and food at home for men and women in the age group 25-34. We calculate the fraction of calories away from home as calories away from home divided by total calories. Finally, from equations (10) and (12), we calculate the steady state weight for men and women with mean age equal to 30. We perform similar experiments for men and women for the remaining age groups from 35-44 to 55-64. We report the results of this experiment for body-mass index and the fraction of calories away from home for men and women in Figures 7 to 10.

For men, changes in food prices and household income account for a large fraction of the increase in body-mass index (all of the increase for age group 25-34 and 55-64 and about 50 percent for age groups 35-44 and 45-54). It accounts for about 50 percent for women younger than 55 and overshoots for women in the age group 55-64.

Changes in food prices and household income account for more than 70 percent of the increase in the calories away from home for women and for men older than 45 and more than hundred percent for men below age 45.

Our results corroborates the existing knowledge on obesity in the following way. On the one hand, economists who use empirical models found that the impact of food prices on weight is small (e.g., Chou et al., 2004, Gelbach et al., 2007, or Chouinard et al., 2007). Using a fully specified calibrated dynamic model, we also find that changes in food prices over time account for almost none of the weight gain by Americans in the last thirty years. On the other hand, researchers in the field of public health (e.g., French, Jeffery, Story, Breitlow, Baxter, Hannan, and Snyder 2001) design small-scale experiments to show that even small changes in food prices can have strong local effect on individual’s food choices. For example, the above-mentioned authors examined the effect of lower prices on sales of lower fat vending machine snacks in 12 work sites and 12 secondary schools. According to
a study, price reductions of ten percent, twenty five percent, and fifty percent on low-fat snacks in vending machines increase the percentage of low-fat snack sales by nine, thirty nine, and ninety three percent, respectively. Using our calibrated dynamic model, we also find that change in food prices affect where people eat (at home or out).
Figure 8: Women’s body-mass index by age

Figure 9: Fraction of calories consumed away from home by men by age
Figure 10: Fraction of calories consumed away from home by women by age
6 Concluding Remarks

In this paper, we further analyzed the impact of changes in food prices and household income on people eating decisions and weight using a dynamic model with rational agents. After careful calibration of the model using evidence from medical research on obesity, we found that food prices determine the allocation of calories across food types, while household income determine the total number of calories consumed and thus individual’s weight. Between 1971 and 2006, changes in food prices alone account for almost none of the change in weight of Americans men and women and for more than fifty percent of the increase in calories consumed away from home. On the other hand, changes in household income account for more than seventy percent of the increase in men’s and women’s weight. Because of the limited effect of food price alone on the body-mass index, we support the view that taxes on food will impact what people eat but will have limited effect on reducing the population body-mass index or the obesity prevalence.

We see two important avenues for academic research on obesity as well as policy recommendation. First, educating people about the benefits of eating healthy, exercising regularly, and the negative health consequences of being obese seem to be promising policies to win the fight against obesity epidemic. Economic research is needed to measure the impact of these education programs on individual’s weight and body-mass index. Second, we derived our results for the impact of food prices on weight and food choices in an environment where agents are fully rational. An alternative view point is that there is nothing optimal in being obese and that individuals experience commitment problems when making food decisions. It is an open and interesting question to revisit the impact of food prices and household income in a set-up where agents have time-inconsistent preferences à la Laibson (1997). We leave these two tasks for future research.
7 Appendix: Deriving first-order conditions in equation (8)

The consumption of food away from home, \(a_t\), and food at home, \(h_t\), appear as follows in the objective function:

\[
\ldots + \beta^{t-1} \pi(W_1) \times \ldots \times \pi(W_{t-1}) \times \\
\left( \pi(W_t) \left[ (\eta a_t^\rho + (1 - \eta) h_t^\rho) \frac{\bar{\pi} (I_t - p_{ht} h_t - p_{at} a_t)^{1-\alpha}}{1 - \sigma} \right] + (1 - \pi(W_t)) \bar{U} \right) + \\
\beta^t \pi(W_1) \times \ldots \times \pi(W_t) \left( \pi(W_t + \lambda(a_t + h_t - c(W_t))) \frac{[(c_{t+1}^f)^\alpha (c_{t+1}^{nf})^{1-\alpha}]^{1-\sigma}}{1 - \sigma} \right) + \\
(1 - \pi(W_t + \lambda(a_t + h_t - c(W_t)))) \bar{U} + \ldots
\]

We take first-order conditions with respect with food away from home, \(a_t\):

\[
\beta^{t-1} \pi(W_1) \times \ldots \times \pi(W_t) [(\eta a_t^\rho + (1 - \eta) h_t^\rho) \bar{\pi} (I_t - p_{ht} h_t - p_{at} a_t)^{1-\alpha}] \times \\
(\eta a_t^\rho + (1 - \eta) h_t^\rho) \bar{\pi} (I_t - p_{ht} h_t - p_{at} a_t)^{1-\alpha} \left( \frac{\alpha \eta a_t^{\rho-1}}{\eta a_t^\rho + (1 - \eta) h_t^\rho} - \frac{p_{at}(1 - \alpha)}{I_t - p_{ht} h_t - p_{at} a_t} \right) + \\
\beta^t \pi(W_1) \times \ldots \times \pi(W_t) \lambda \pi'(W_{t+1}) \left( \frac{[(c_{t+1}^f)^\alpha (c_{t+1}^{nf})^{1-\alpha}]^{1-\sigma}}{1 - \sigma} - \bar{U} \right) = 0
\]

Simplifying the previous expression, we get:

\[
(1 - \sigma) U(c_t^f, c_t^{nf}) \left( \frac{\alpha \eta a_t^{\rho-1}}{\eta a_t^\rho + (1 - \eta) h_t^\rho} - \frac{p_{at}(1 - \alpha)}{I_t - p_{ht} h_t - p_{at} a_t} \right) + \beta \lambda \pi'(W_{t+1})(U(c_{t+1}^f, c_{t+1}^{nf}) - \bar{U}) = 0
\]

(22)

Similarly, after taking first-order condition with respect to \(h_t\), we get:

\[
(1 - \sigma) U(c_t^f, c_t^{nf}) \left( \frac{\alpha (1 - \eta) h_t^{\rho-1}}{\eta a_t^\rho + (1 - \eta) h_t^\rho} - \frac{p_{ht}(1 - \alpha)}{I_t - p_{ht} h_t - p_{at} a_t} \right) + \beta \lambda \pi'(W_{t+1})(U(c_{t+1}^f, c_{t+1}^{nf}) - \bar{U}) = 0
\]

(23)

Rearranging the previous two conditions, we obtain the system of equations (8):

\[
\frac{\alpha \eta a_t^{\rho-1}}{\eta a_t^\rho + (1 - \eta) h_t^\rho} - \frac{p_{at}(1 - \alpha)}{I_t - p_{at} a_t - p_{ht} h_t} = - \frac{\lambda \beta}{1 - \sigma} \pi'(W_{t+1}) \frac{U(c_{t+1}^f, c_{t+1}^{nf}) - \bar{U}}{U(c_t^f, c_t^{nf})}
\]

(24)

\[
\frac{\alpha (1 - \eta) h_t^{\rho-1}}{\eta a_t^\rho + (1 - \eta) h_t^\rho} - \frac{p_{ht}(1 - \alpha)}{I_t - p_{at} a_t - p_{ht} h_t} = - \frac{\lambda \beta}{1 - \sigma} \pi'(W_{t+1}) \frac{U(c_{t+1}^f, c_{t+1}^{nf}) - \bar{U}}{U(c_t^f, c_t^{nf})}
\]
References


