The Risk of Hitting the Zero Lower Bound and the Optimal Inflation Target

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Abstract

Based on the US data on interest rates, I find that the risk for the nominal interest rate to hit the zero lower bound (ZLB) is around 16% in the US if the Fed continues to keep the inflation target at 2%. I then develop a small dynamic stochastic general equilibrium model featuring an occasionally binding ZLB, and calibrate the model to match with the risk. Solving the model using a fully nonlinear method, I find that the optimal inflation target is around 3.5%. In addition, the optimal inflation target is sensitive to both the risk of hitting the ZLB and the degree of inflation indexation.

JEL classification: E52, E58.

Keywords: Risk of hitting the ZLB; Optimal inflation rate; Inflation indexation; ZLB; Rotemberg pricing scheme.

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1 Introduction

Inflation targeting is very important monetary policy strategy that helps create a nominal anchor to tie down the price level, so central bankers are able to obtain their price stability objectives. Since the early 1990s, advanced economies started using this strategy, either explicitly, i.e. Canada and the UK, or implicitly, i.e. the US. Although different countries pursue different inflation rates, the conventional inflation target is around 2% for the advanced economies.

However, since the late 1990s when Japan fell into the liquidity trap with binding ZLB, economists, such as Krugman (1998), have debated whether central banks should raise their inflation targets above the status quo of 2% and what the optimal inflation target should be. These topics are even more important today as the US target federal funds rate has reached the zero bound since December 2008 and the US economy experienced its greatest slump since the Great Depression. Prominent economists, including Blanchard et al. (2010) and Ball (2013), suggested that policymakers might consider an inflation target of around 4%.

The suggestion lies under the argument that, in the presence of the ZLB, a significantly higher inflation target results in higher expected inflation and, as a result, higher nominal interest rates that create leeway for the central bank to deal with a particularly adverse demand shock. Specifically, when the nominal interest rate reaches the ZLB, monetary policy is more expansionary in an economy with a higher inflation target. However, higher inflation is always associated with higher adjustment costs if firms do not foresee their future optimal prices, which is always the case, and want to adjust their prices. In addition, higher inflation might generate a larger inefficiency wedge due to price distortion if firms are not able to charge their optimal prices.

This paper aims at answering the question what inflation target is optimal in a fully nonlinear dynamic stochastic general equilibrium (DSGE) model with an occasionally binding zero lower bound (ZLB) on the nominal interest rate. To this end, I first carefully compute the risk, or the unconditional probability, of hitting the ZLB using the US data on interest rates. I then develop a New Keynesian model, calibrate the model to match with the risk, and solve it using a fully nonlinear method.

I find that, based on the US data on interest rates, the unconditional probability of hitting the ZLB in the US would have been 16.1% had the Fed targeted the inflation rate of 2%. Calibrating the model to match with this risk and the inflation indexation found in empirical studies, I find that the optimal inflation rate is around 3.5%, which is slightly smaller than the value suggested by Blanchard et al. (2010) and Ball (2013). The optimal inflation is sensitive to both the risk of hitting the ZLB and the degree of inflation indexation.

The related literature includes Schmitt-Grohe and Uribe (2010), Billi (2011), Coibion et al. (2012), and Ngo (2014). Except Ngo (2014), these papers do no use any fully nonlinear method. All the papers are also different from the current paper in many aspects. In Schmitt-Grohe and Uribe (2010), the central bank is able to commit to policy plans and the unconditional probability of hitting the ZLB is 0%. They find that the optimal inflation rate is around 0%. Ngo (2014) also finds an optimal inflation rate of 0%. However, the central bank in his model conducts discretionary monetary policy instead of a Taylor rule, as in this paper. In addition, Ngo (2014) does not allow inflation indexation in his model and the unconditional probability of hitting the ZLB is much lower than the probability we observed in the US data.

Coibion et al. (2012) find that the optimal inflation rate is around 1.5% in their benchmark model. They calibrate the model such that the unconditional probability of hitting the ZLB is about 5% post World War II, or approximately three years out of sixty years. This unconditional probability is much smaller than what I compute in this paper based on the US data on interest rates. The reason is that, to compute the risk of hitting the ZLB, Coibion et al. (2012) use a shorter time series of federal funds target rates. Specifically, they assume that the ZLB would end in 2011. In addition, they do not address potential biases that might arise when they use the nominal interest to compute the risk of hitting the ZLB. Moreover, I allows a higher degree of inflation indexation that matches with empirical studies, and I use the Rotemberg price scheme instead of the Calvo pricing method as in their paper.

In contrast, Billi (2011) finds that, in the case of a Taylor rule, the optimal inflation target is about 8%, which is larger than the optimal inflation rate found in this paper. In his paper, Billi (2011) also allows inflation indexation. However, he uses the past inflation as a proxy for the expected inflation. In this paper, I use the inflation target as the proxy for the expected inflation.[1] As a result, deflationary/disinflationary episodes are less persistent in my model, and the central bank pursues a smaller inflation target.

Another difference between this paper and the other two papers, Billi (2011) and Coibion et al. (2012), is that I solve a fully nonlinear model instead of using an approximation of the model. The nonlinearity might cause the results to be different.

The remainder of this paper is organized as follows. Section 2 shows how to compute the risk of hitting the ZLB using the US data. Section 3 presents the structure of the model. Section 4 shows the benchmark calibration and solution method, and Section 5 presents main results. Section 6 contains some sensitivity analyses of the main findings with respect to some important assumptions. Section 7 concludes.

2 The risk of hitting the ZLB

The risk of hitting ZLB plays a key role in determining the optimal inflation target. Specifically, when the risk of hitting the ZLB is high, the expected cost of hitting the ZLB is high

[1] Empirical studies, such as Ascari et al. (2011), find that firms use both the past inflation and the inflation target to index their prices. It is ideal to investigate both of them at the same time. However, to easily compare with the ZLB literature and to improve numerical efficiency, I decide to use only the inflation target for inflation indexation. Studying both the past inflation and the inflation target for inflation indexation would be interesting and is part of my future research agenda.

given the fact that the ZLB is very damaging. Many economists, including Mishkin (2011), believe that the 2007-2009 recession with a binding ZLB is a rare disaster that occurs once every seventy years. However, other economists, i.e. Ball (2013), disagree. In this section, I carefully compute the risk that the nominal interest rate hits the ZLB if the Fed continues to keep the inflation target at 2%.

Panel A of Figure 1 shows the target and actual (or effective) federal funds rates in the US since 1981:I, the data for the target federal funds rate was not available until 1982:III. These federal funds rates are tightly correlated and they have reached the ZLB since December 2008 when the economy was in the middle of the 2007-2009 recession.[2] By the end of 2014, the ZLB duration had been 25 quarters, which is much longer than the existing ZLB literature suggests.

If we use the time series data of the target federal funds rate since the data was first available in September 1982, the unconditional probability of hitting the ZLB is 19.2%, or 25 quarters out of 130 quarters. Moreover, if we use the data from the first quarter of 1990, when some advanced economies, including the US, started pursuing the inflation targeting strategy either explicitly or implicitly, the probability is around 26%, or 25 quarters out of 96 quarters. This value is considered the upper bound for the risk of hitting the ZLB.

These unconditional probabilities overstate the actual risk of hitting the ZLB in the US because I did not use all the data available for the actual federal funds rate. If I use the data for the actual federal funds rate since the data was first available in June 1954, the total number of observations is 244 quarters. So the unconditional probability of hitting the ZLB is around 10.3%. However this number definitely understates the actual risk of hitting the ZLB and is considered as the lower bound of the unconditional probability of hitting the ZLB. The reason is that, not until the very early 1990s, the Fed and other central banks

[2]To be more precise, the effective federal funds rate has been very close to zero, around 0.12% on average, during this period.

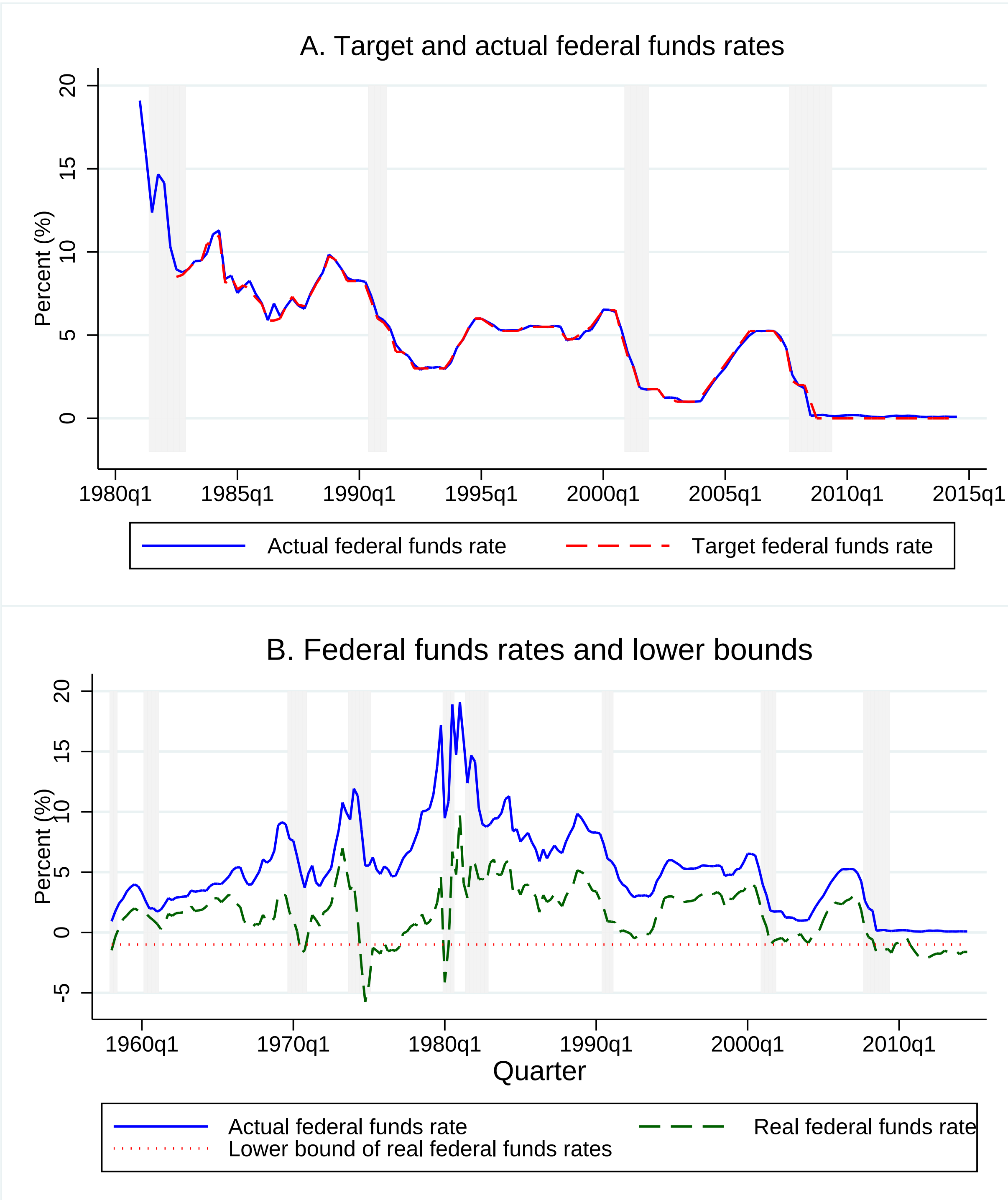

Figure 1: Real federal funds rate is the actual federal funds rate minus the inflation rate computed as a percentage change in the CPI of All Items Less Food and Energy a year ago. The shaded areas indicate the US recessions. Source: the Federal Reserve Economic Data.

started pursuing the inflation target of 2%, either explicitly or implicitly. Therefore, before 1990 it is likely that the actual inflation rate and the implicit target inflation rate might have been greater than 2%, making 10.3% smaller than the actual risk.

To address the bias of these unconditional probabilities, we should answer the question raised in Ball (2013): what would have the unconditional probability of hitting the ZLB been, had the Fed targeted the inflation rate of 2%. To this end, I follow Ball (2013) and use the real interest rate to answer the question. Specifically, the nominal interest rate equals the real interest rate plus the inflation rate. Therefore, we can interpret the zero lower bound on the nominal interest rate as a bound of minus inflation for the real interest rate. If the target inflation rate is 2%, the inflation rate is 2% and the bound on the real interest rate is -2%. However, Ball (2013) argues that a recession is likely to push inflation down somewhat and that the history suggests that the inflation fell about 1% during the past recessions that started with $2-3\%$ inflation rates. Therefore he finds that the bound on the real interest rate is -1%.

Panel B of Figure 1 shows: (i) the actual federal funds rate; (ii) the real interest rate computed as the actual federal funds rate minus the inflation rate, where the inflation rate is calculated as a percentage change of the CPI of All Items Less Food and Energy from a year ago; and (iii) the lower bound of the real interest rate. The data spans from 1957:IV, when the data for the CPI of All Items Less Food and Energy was first available, to 2014:IV. So we have 229 observations in all.

From Panel B we are able to see that the real interest rate was smaller than the bound, and, as a result, the nominal interest rate might have hit the ZLB, in the four recessions: 1957:III-1958:II, 1969:IV-1970:IV, 1973:IV-1975:I, 1980:I-1980:IV, and 2007:IV-2009:II.[3] Es-

[3]Ball (2013) argues that in the three out of 7 recent recessions excluding the 2007-2009 recession, the nominal interest rate would have hit the ZLB if the inflation rate had been around 2% at the start of the recessions. These three recessions include the 1969-1970 recession, the 1973-1975 recession, and the 1980 recession. Hence, the probability of hitting the ZLB conditional on a recession is around 50%, or four recessions out of eight recessions.

pecially, using the real interest rate we can very well infer that the nominal interest rate reached the ZLB during the 2007-2009 recession. In addition, the nominal interest rate almost hit the ZLB in the 2001 recession.

Examining the real interest rate since 1957:IV when the CPI data was first available, I find that the ZLB was binding in 37 quarters. Given that the sample has 229 quarters, the unconditional probability of hitting the ZLB is 16.1%. For a robust check, if I use the CPI of All Items instead of the CPI of All Items Less Food and Energy, the result slightly changes. In particular, the unconditional probability of hitting the ZLB increases to about 17.2%, and the sample is longer with 243 observations. However, using the CPI of All Items results in a binding ZLB in some periods when the economy was not in a recession. The reason is that the inflation rate based on all items is more volatile. The result that the ZLB binds during a normal time is hard to justify. Therefore, I will calibrate shocks in this paper to match with 16.1%, the unconditional probability of hitting the ZLB would have been if the Fed had targeted an inflation rate of 2%.

Another robust check is to raise the lower bound on the real interest rate. The reason is that, using the real interest rate slightly underestimates the actual ZLB as the real interest rate was larger than the bound in 2009:IV and early 2010. This occurs because the inflation rate fell more than 1%, the decrease that Ball (2013) assumes in order to find out the bound for the real interest rate. If we allowed the inflation rate to fall more than 1% in a recession, i.e. 1.5%, then the lower bound on the real interest rate would be -0.5% if the Fed targeted a 2% inflation rate. Using this new lower bound, the risk of hitting the ZLB would increase to 20.6%. However, the ZLB was binding in many periods when the economy was in normal time.

The unconditional probability of 16.1% is between the possible range $10.3\% - 26\%$ that I computed above, and is greater than any value used in the existing ZLB literature, including Coibion et al. (2012). For example, they calibrate the model such that the unconditional probability of hitting the ZLB is three years out of sixty years, or around 5%, if the target inflation rate is around 3.5%. According to this calibration, the unconditional probability of hitting the ZLB would be only 10% if the inflation target was 2%.

3 Model

The model consists of a continuum of identical households, a continuum of identical competitive final good producers, a continuum of monopolistically competitive intermediate goods producers, and a monetary authority.

3.1 Households

The representative household maximizes his expected discounted utility

$$Max\ E_t \left\{ \left(\frac{C_t^{1-\gamma}}{1-\gamma} + \chi \frac{N_t^{1+\eta}}{1+\eta} \right) + \sum_{j=1}^{\infty} \left\{ \beta^j \left(\prod_{k=0}^{j-1} \beta_{t+k} \right) \left(\frac{C_{t+j}^{1-\gamma}}{1-\gamma} + \chi \frac{N_{t+j}^{1+\eta}}{1+\eta} \right) \right\} \right\} \tag{1}$$

subject to the budget constraint

$$C_t + B_t = w_t N_t + B_{t-1} \left(\frac{1+i_{t-1}}{1+\pi_t} \right) + \int_0^1 D_t(i) di + T_t, \tag{2}$$

where C, N are composite consumption and total labor; B, D, T denote real bonds, dividends, and lump sum transfers; i, π are the nominal interest rate and the inflation rate, respectively; w is the real wage; γ, η, χ are the risk aversion parameter, the inverse wage elasticity of labor with respect to wages, and the steady state labor determining parameter; β_t is the shock to the subjective time discount factor β, or the preference shock, that follows an AR(1) process

$$\ln\left(\beta_{t+1}\right) = \rho_\beta \ln\left(\beta_t\right) + \varepsilon_{\beta,t+1}, \text{ where } \beta_t \text{ is given,} \tag{3}$$

where $\rho_\beta \in (0,1)$ is the persistence of the preference shock; and $\varepsilon_{\beta t}$ is the innovation of the preference shock with mean 0 and variance σ_β^2. The preference shock is a reduced form of more realistic forces that can drive the nominal interest rate to the ZLB.

The optimal choices of the household give rise to the labor supply

$$\chi N_t^\eta C_t^\gamma = w_t, \tag{4}$$

and the Euler equation

$$E_t\left(M_{t,t+1}\left(\frac{1+i_t}{1+\pi_{t+1}}\right)\right) = 1, \tag{5}$$

where $w_t = W_t/P_t$ is the real wage, $\pi_t = P_t/P_{t-1} - 1$ is the inflation rate, and the stochastic discount factor is given by

$$M_{t,t+1} = \beta\beta_t\left(\frac{C_{t+1}^{-\gamma}}{C_t^{-\gamma}}\right). \tag{6}$$

3.2 Final goods producers

To produce the composite final goods, the final goods producers buy and aggregate a variety of intermediate goods using a CES technology. Their cost-minimization problem is given below.

$$\min \int_0^1 P_t(i)\, Y_t(i)\, di \text{ s.t. } Y_t = \left(\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}}\, di\right)^{\frac{\epsilon}{\epsilon-1}}, \tag{7}$$

where $P_t(i)$ and $Y_t(i)$ are the price and the amount of intermediate goods $i \in [0,1]$; and ε is the elasticity of substitution among intermediate goods.

The optimal condition gives rise to the demand for the intermediate goods i

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} Y_t, \tag{8}$$

and the aggregate price level

$$P_t = \left(\int P_t\,(i)^{1-\epsilon}\,di \right)^{\frac{1}{1-\epsilon}}. \tag{9}$$

3.3 Intermediate goods producers

There is a mass one of intermediate goods producers that are monopolistic competitors. Given its price $P_t(i)$ and demand $Y_t(i)$, firm $i \in [0,1]$ chooses labor that

$$\min \{w_t N_t(i)\} \qquad s.t. \qquad Y_t(i) = A_t N_t(i), \tag{10}$$

where A denotes the technology shock; $\rho_A \in (0,1)$ is the persistence of the technology shock; and ε_{At} is the innovation of the technology shock with mean 0 and variance σ_A^2.

Let $\varphi_{i,t}$ be the Lagrange multiplier with respect to the production. The first-order condition gives the same marginal cost, φ_t, to all firms:

$$\varphi_t = \varphi_{i,t} = \frac{w_t}{A_t}. \tag{11}$$

3.4 Price adjustments

The intermediate goods firms adjust their prices according to Rotemberg (1982). Specifically, they have to pay an adjustment cost in terms of final goods when they change their prices. Following Aruoba and Shorfheide (2013) and allowing some degree of inflation indexation, the problem of firm i, for $i \in [0,1]$, is given as follows:

$$\max_{\{P_t(i)\}} E_t \sum_{j=0}^{\infty} \left\{ M_{t,t+j} \left[\left(\frac{P_{t+j}(i)}{P_{t+j}} - \varphi_t \right) Y_{t+j}(i) - \frac{\varphi}{2} \left(\frac{P_{t+j}(i)}{P_{t+j-1}(i)} - (1+\theta\pi) \right)^2 Y_{t+j} \right] \right\} \tag{12}$$

subject to its demand in equation (8) and

$$M_{t,t+j} = 1 \text{ if } j = 0; M_{t,t+j} = \prod_{s=0}^{j-1} M_{t+s,t+s+1} \text{ for } j \geq 1, \tag{13}$$

where φ is the adjustment cost parameter, π is the target inflation, and θ is inflation indexation. According to this formulation, the firm is allowed to index its price to the expected inflation, which is the same as the target inflation rate set by the central bank, and it only pays adjustment costs if its new price is different from the indexed price, $(1+\theta\pi)P_{i,t-1}$.

In a symmetric equilibrium, all firms will choose the same price and produce the same quantity, i.e. $P_t(i) = P_t$ and $Y_t(i) = Y_t$. The optimal pricing rule then gives rise to the following condition:

$$\left(1 - \varepsilon + \varepsilon \frac{w_t}{A_t} - \varphi(\pi_t - \theta\pi)(1+\pi_t)\right) Y_t + \varphi E_t \left[M_{t,t+1}(\pi_{t+1} - \theta\pi)(1+\pi_{t+1}) Y_{t+1}\right] = 0, \tag{14}$$

where $M_{t,t+1}$ is the stochastic discount factor defined in equation (6).

3.5 Monetary policy

The central bank conducts monetary policy using a simple Taylor rule as follows:

$$\left(\frac{1+i_t}{1+i}\right) = \left(\frac{Y_t}{Y}\right)^{\phi_y} \left(\frac{1+\pi_t}{1+\pi}\right)^{\phi_\pi}, \tag{15}$$

$$i_t \geq 0, \tag{16}$$

where π, i, Y are the targeted inflation, the steady state nominal interest rate, and the steady state output, respectively.

Equation (16) implies that the nominal interest rate is not allowed to be negative. This is the key condition in the ZLB literature.

3.6 Aggregate condition and equilibrium

$$Y_t = A_t N_t, \tag{17}$$

and the resource constraint is given by

$$C_t + \frac{\varphi}{2} (\pi_t - \theta\pi)^2 Y_t = Y_t. \tag{18}$$

The equilibrium system consists of a system of six nonlinear difference equations (4), (5), (14), (15), (17), (18) together with the ZLB (16) for six variables w_t, C_t, i_t, π_t, N_t, and Y_t.

4 Solution method and calibration

4.1 Solution method

Following the method used in Ngo (2014), I solve the model using a collocation method associated with cubic spline basis functions to capture kinks due to the ZLB. At each collocation node, I solve a complementarity problem using the Newton method and the semi-smooth root-finding algorithm as described in Miranda and Fackler (2002). I also use an analytical Jacobian matrix computed from the approximating functions. Moreover, I write the code using a parallel computing method that allows me to split up a large number of collocation nodes into smaller groups that are then assigned to different processors to be solved simultaneously. This procedure reduces computation time significantly.[4]

[4] I obtain the maximal absolute residual across the equilibrium conditions of the order of 10^{-8} for almost all states off the collocation nodes. For a few off-collocation states when the ZLB becomes binding, the maximal absolute residual is of the order of 10^{-5}.

4.2 Calibration

I calibrate the parameters on the basis of the observed data and other studies. The quarterly subjective discount factor β is set at 0.997, as in Woodford (2011). The constant relative risk aversion parameter γ is 1, corresponding to a log utility function with respect to consumption. This utility function is commonly used in the literature of business cycles. The labor supply elasticity with respect to wages is set at 1, or $\eta = 1$, as in Woodford (2011). I set the parameter associated with labor preference $\chi = 1$.[5] The elasticity of substitution among differentiated intermediate goods ϵ is 7.66, corresponding to a 15% net markup that is in the range found by Diewert and Fox (2008). This value is also popular in the literature (e.g., Adam and Billi (2007) and Braun et al. (2013)).

The price adjustment cost parameter, φ, is calibrated to be 132 corresponding to the probability of keeping prices unchanged of 0.8, which is in the range estimated by Christiano et al. (2005) and is still smaller than the value used in Christiano et al. (2011), 0.85.[6] Some authors, including Nakata (2011) and Ascari et al. (2011), use/estimate higher values for the price adjustment cost parameter. However, using these higher values would result in a longer duration of keeping prices unchanged that is not in line with empirical studies, see Nakamura and Steinsson (2008) for more detailed discussion.

[5] As in Fernandez-Villaverde et al. (2012), this parameter does not affect the results of the paper significantly.

[6] This comparison is up to the first-order approximation around the steady state inflation of 0%, see Miao and Ngo (2014) for more discussion.

Table 1. Benchmark Parameterization

Symbol	*Description*	*Values*
β	Quarterly discount factor	0.997
γ	CRRA parameter	1
η	Inverse labor supply elasticity	1
ε	Monopoly power	7.66
θ	Inflation indexation	0.9
φ	Price adjustment cost parameter in the Rotemberg model	132
π	Inflation target, 2% per year	0.005
ϕ_π	Weight of target inflation in the Taylor rule	1.5
ϕ_y	Weight of output target in the Taylor rule	0.125
ρ_β	AR-coefficient of preference shocks	0.65
σ_β	Standard deviation of the innovation of preference shocks (%)	0.5

I set the parameters in the Taylor rule $\phi_\pi = 1.5$ and $\phi_y = 0.125$, as in Gali (2008) and Ascari and Rossi (2012), which are conventional in the literature. I set the inflation indexation, θ, at 0.9, which is in the range estimated by Ascari et al. (2011) and is the same degree of inflation indexation used in Billi (2011). Because this parameter is important, I also conduct a sensitivity analysis regarding the parameter in a section below.

In this paper, I shut down the technology shock in all numerical solutions in order to focus on the preference shock that is considered as the main force driving the economy to the ZLB.[7] Following Nakov (2008), I set the persistence of the preference shock, ρ_β, at 0.65, which reflects the persistence of the natural rate of interest rate. Nakov (2008) argues that this value is between 0.35 used by Woodford (2003) and 0.8 used by Adam and Billi (2007).

[7]Guerrieri and Lorenzoni (2011) show that the preference shock is the reduced form of the deleveraging shock, a shock to debt limits that causes the nominal interest rate to reach the ZLB.

The remaining and the most difficult task is to determine how large the standard deviation of the innovation of preference shock actually is.

In this paper, I calibrate the standard deviation of the innovation of preference shocks to be 0.5% per quarter, which enables the model to generate the unconditional probability of hitting the ZLB of 16.1% that I computed in Section 2. As I discuss above, this unconditional probability of hitting the ZLB is greater than any value used in the existing literature, including Fernandez-Villaverde et al. (2012). I also implement a sensitivity analysis for the results of this paper regarding the risk of hitting the ZLB in Section 6.

5 Results

To find out the optimal target inflation rate, I first solve the model to find the value function and the policy function with respect to different values of the inflation target. I then take a random sample of $99,999$ preference shocks, and compute the unconditional (average) welfare based on the value function found in the first step. Eventually, I compute welfare gain as a percentage change of the unconditional welfare from the one associated with the conventional inflation target of 2% that the Fed has pursued implicitly. In addition, I compute the long-run inflation corresponding to each value of the inflation target, which is the average inflation rate from a simulation of $99,999$ periods starting from the steady state. I also compute the unconditional probability of hitting the ZLB based on the $99,999$-period simulation, which is the ratio between the number of periods with binding ZLB and $99,999$ periods simulated. The results are shown in Figure 2.

The solid red line in Figure 2, with the y-axis on the right, presents the unconditional welfare gain. Apparently, the welfare gain first increases with the inflation target and reaches the highest value of around 0.065% at the inflation target of about 3.5% before decreases with the inflation target. The intuition is that: when the inflation target is higher, both

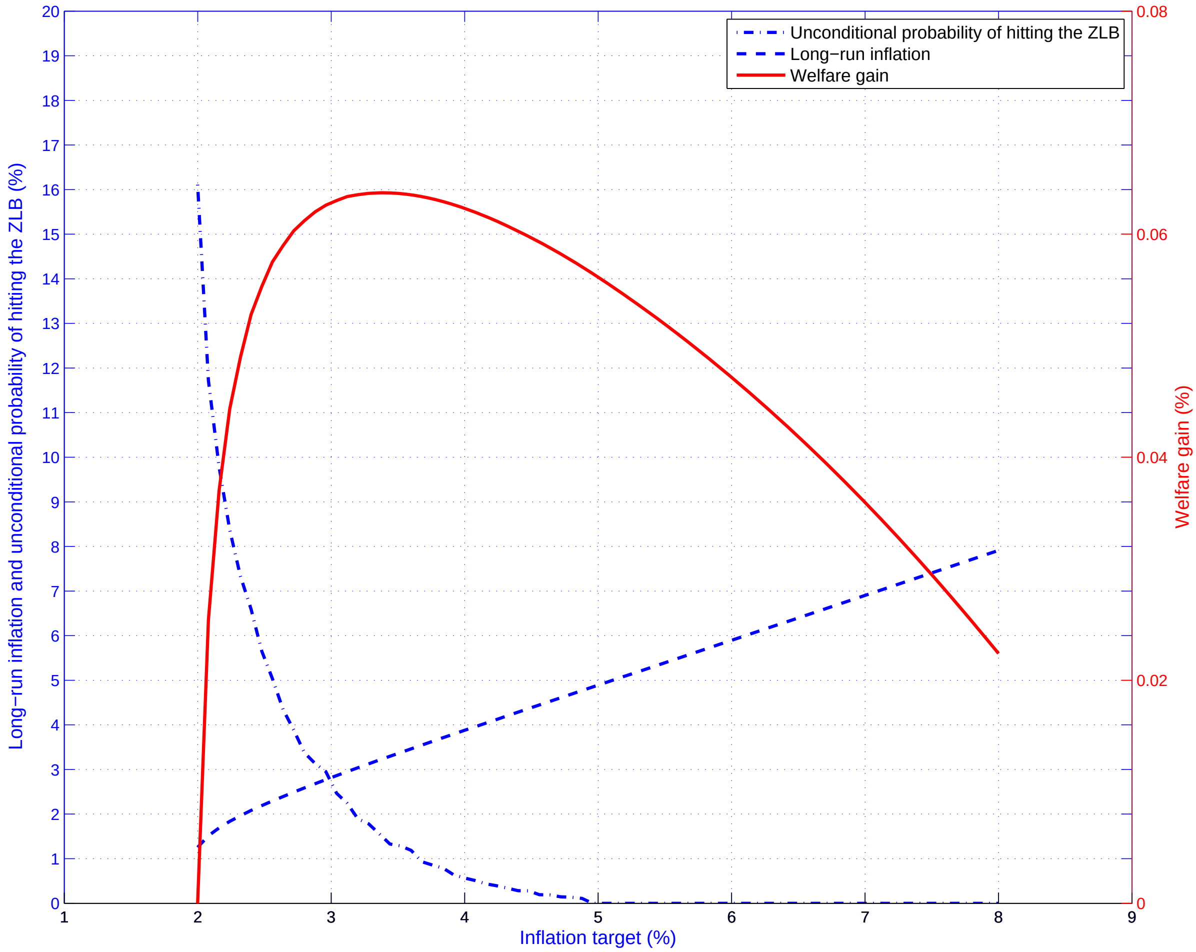

Figure 2: Welfare gain, long-run inflation, and unconditional probability of hitting the ZLB. Welfare gain is a percentage change in unconditional welfare due to targeting an inflation rate greater than the conventional target of 2%. Unconditional welfare is the average welfare based on a sample of 99,999 preference shocks. Long-run inflation is the average inflation rate based on a simulation of 99,999 periods starting from the steady state. Unconditional probability of hitting the ZLB is the ratio between the number of periods with a binding ZLB and 99,999 periods simulated.

the probability of hitting the ZLB and output losses are smaller. As a result, the benefit of targeting an inflation rate higher than 2% is more than to offset the cost caused by the higher inflation. Therefore, the welfare gain increases. However, when the inflation target is much higher than 2%, the cost of the higher inflation outweighs the benefit and the welfare gain decreases. In conclusion, the optimal target inflation rate is about 3.5% per year.

As shown in Figure 2 with the y-axis on the left, the higher the inflation target, the higher the long-run inflation, and the lower the probability of hitting the ZLB. For example, if the inflation target is 2%, the long-run inflation is around 1.3% and the probability of hitting the ZLB is around 16.1%. When the inflation target increases to 3.5%, the long-run inflation is around 3.5% and the unconditional probability of hitting the ZLB reduces to around 1.3%.

The optimal inflation target of 3.5% is between ones found in Coibion et al. (2012) and Billi (2011). Specifically, Billi (2011) finds that, in the case of a Taylor rule, the optimal inflation target is as big as about 8%. The main reason for the difference is that Billi (2011) uses the last inflation for inflation indexation. Due to this characteristic, under a particularly adverse shock that causes the ZLB to bind, his model generates very persistent deflationary/disinflationary episodes associated with binding ZLB and output losses. Consequently, a binding ZLB is very damaging in his model. Hence, the central bank pursues a very high inflation target. In his framework, the probability of hitting the ZLB is approximately zero under the optimal inflation target of 8%.

Instead of using the past inflation as the benchmark for inflation indexation as in Billi (2011), in this paper I use the inflation target for indexing inflation. By doing so, the model in this paper does not produce very persistent deflationary/disinflationary episodes associated with binding ZLB. Therefore, a binding ZLB is less damaging in this paper than in Billi (2011), and the optimal inflation target is smaller than the one found in his paper. Note that the degree of inflation indexation of 0.9 in this paper is similar to the one used in

Billi (2011).

The optimal inflation target in the benchmark model of Coibion et al. (2012) is around 1.5% per year and smaller than the one in this paper. There are several reasons for the discrepancy. First, I model the price adjustment using adjustment costs, as in Rotemberg (1982), instead of using the time-dependent pricing, as in Calvo (1983). Second and more importantly, I calibrate the preference shock such that the unconditional probability of hitting the ZLB matches with what we observed in the US post World War II. This probability is much higher than the one used in the benchmark model of Coibion et al. (2012), making both conditional and unconditional cost of the ZLB higher.

In addition, the degree of inflation indexation in this paper is 0.9, which is in the range estimated by Ascari et al. (2011) and is the same value used in Billi (2011). This degree of inflation indexation is greater than the one used in Coibion et al. (2012). As a result, the cost of high steady state inflation incurred every period is smaller in this paper than in their paper. In total, the net benefit of targeting a significantly positive inflation rate is greater in this paper. Another reason that might explain the discrepancy is that I solve a fully-nonlinear model instead of using the first-order approximation to the FOCs and the second-order approximation to the utility function, as in Coibion et al. (2012).

6 Sensitivity analyses

For a robustness check, I redo the exercise for different values of the risk of hitting the ZLB and the degree of inflation indexation.

6.1 The risk of hitting the ZLB

To see how sensitive the optimal inflation target is to the risk of hitting the ZLB, I first recalibrate the volatility of the innovation of the preference shock such that the unconditional probability of hitting the ZLB is 5% and 10%. For each case, I then resolve the model and find out the optimal inflation target. The results are presented in Panel A of Figure 3.

Apparently, when the risk of hitting the ZLB reduces, the optimal inflation target declines. The optimal inflation target is only 2.6% when the risk of hitting the ZLB is 5%, and 3.1% when the risk is 10%. Intuitively, when the risk of hitting the ZLB decreases, the expected cost of hitting the ZLB decreases. Therefore, it is better for the central bank to lower the optimal inflation target.

6.2 Inflation indexation

In this subsection, I redo the computation for two cases: the inflation indexation is 0.85 and 1 instead of 0.9 as in the benchmark calibration. In these case, I recalibrate the standard deviation of the innovation of preference shocks such that the unconditional probability of hitting the ZLB remains unchanged, about 16%. The results are presented in Panel B of Figure 3.

Panel B of Figure 3 shows that for the case when the inflation indexation is 0.85, the optimal target inflation is around 3% with the maximal welfare gain of about 0.05%. For the case when the inflation is fully indexed, the optimal target inflation is around 5%.

Intuitively, when the inflation indexation is higher, the marginal cost of raising the inflation target is smaller because firms' prices are closer to the optimal price even if the firms do not adjust their prices. In the meantime, with a higher inflation target, the probability of hitting the ZLB decreases, and the risk of falling into a deep recession with a binding ZLB is smaller. Therefore, it is better for the central bank to raise the inflation target.

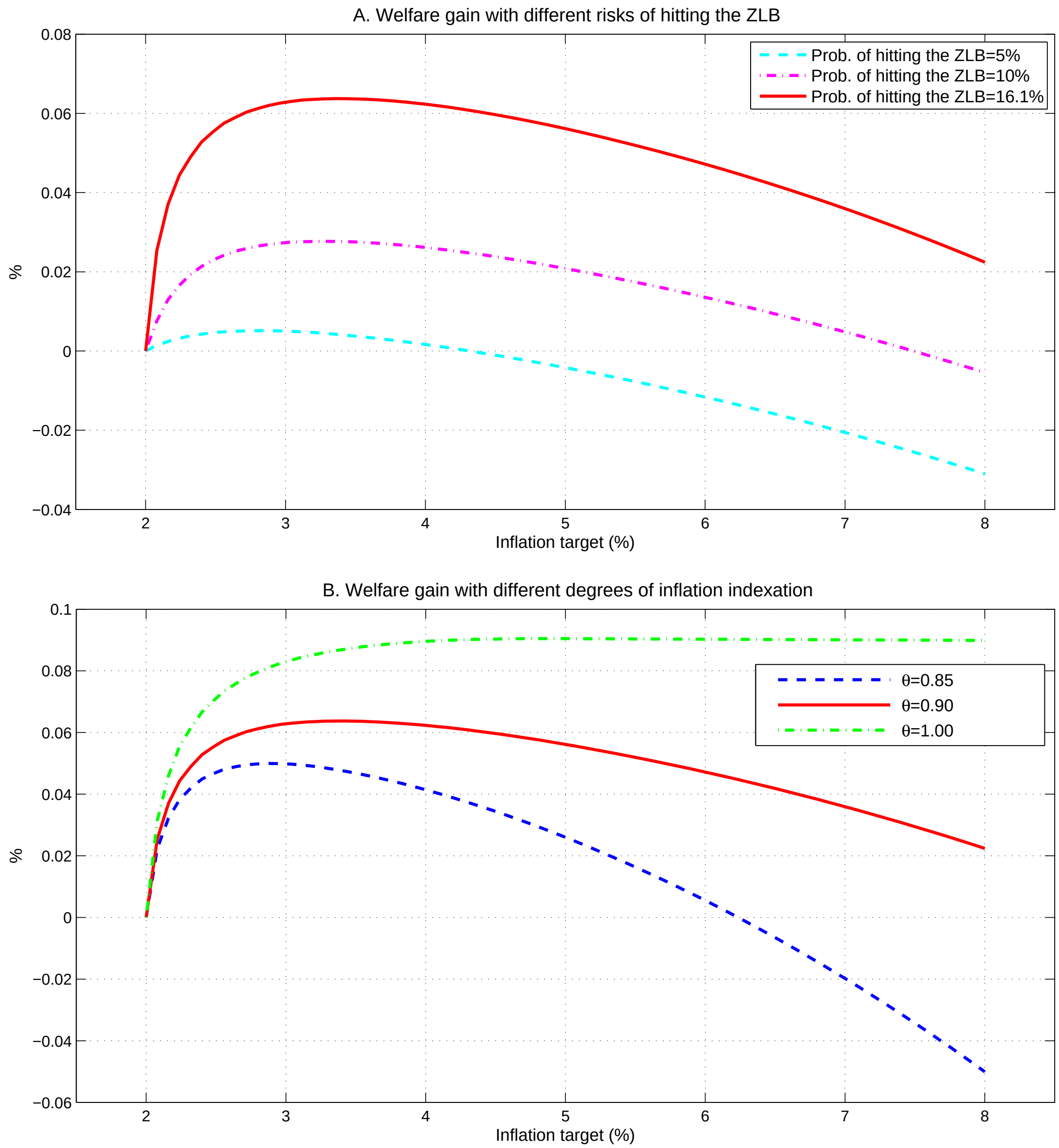

Figure 3: Sensitivity analysis. Welfare gain is a percentage change in unconditional welfare due to targeting an inflation rate greater than the conventional target of 2%. Unconditional welfare is the average welfare based on a sample of $99,999$ preference shocks.

7 Conclusion

This paper investigates what inflation target is optimal in the New Keynesian framework featuring an occasionally binding ZLB on the nominal interest rate. Solving the fully non-linear model using a global method, I find that, under the calibration that matches with the unconditional probability of hitting the ZLB in the US and with the inflation indexation found in empirical studies, the optimal inflation rate is around 3.5%. The optimal inflation target is sensitive to both the risk of hitting the ZLB and the degree of inflation indexation. When prices are fully indexed by the target inflation rate, the optimal inflation target rises to around 5%.

Apparently, there are several ways in which we can extend this paper. First, in this paper, firms adjust their prices using the Rotemberg pricing scheme, which is just a simple menu cost model. It would be interesting to see if the result changes under different pricing schemes, such as the Calvo time-dependent pricing scheme or the state dependent pricing scheme of Dotsey et al. (1999) or Gertler and Leahy (2008). Second, it would be interesting to see if adding more realistic features, such as habit formation, policy inertia, and inflation inertia, would change the result significantly.[8]

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[8]Studying the impact of different pricing schemes and these features in a fully-nonlinear framework as in this paper is part of my future research agenda.

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