Advanced Optics Laboratory

The Anomalous Zeeman Splitting of the Sodium 3P States

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Objectives

- Calibrate Fabry-Perot interferometer
- Determine the Zeeman splitting of the 3P energy state of the Sodium atom
- Determine effective nuclear charge of sodium atom
- Determine strength of internal magnetic field

Zeeman Effect - Historical Origin

- Zeeman Effect is named after Dutch Physicist, Pieter Zeeman.
- The experimental Evidence of this effect was published in:

i) P. Zeeman, "*The Effect of Magnetisation on the Nature of Light Emitted by a Substance*" Nature 55: 347. (1897)

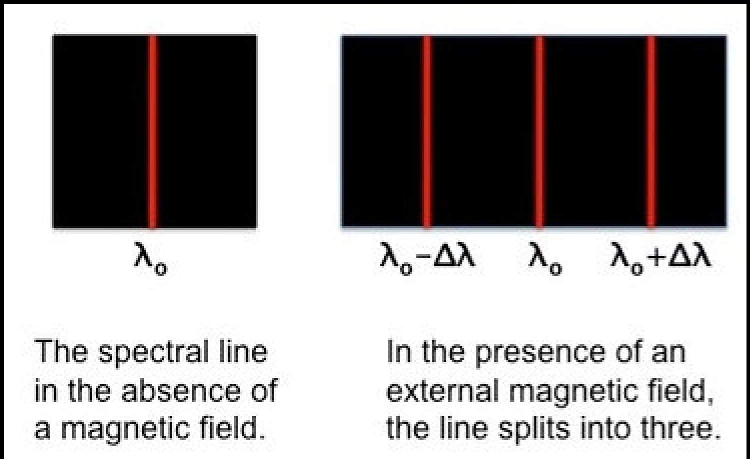
ii) P. Zeeman, "*Doubles and triplets in the spectrum produced by external magnetic forces*". Phil. Mag. 44: 55. (1897)

- He obtained the Nobel Prize for Physics in 1902.

Dr. Pieter Zeeman

Zeeman Effect

- Zeeman Effect is the splitting of spectral lines into multiple lines in the presence of a static magnetic field

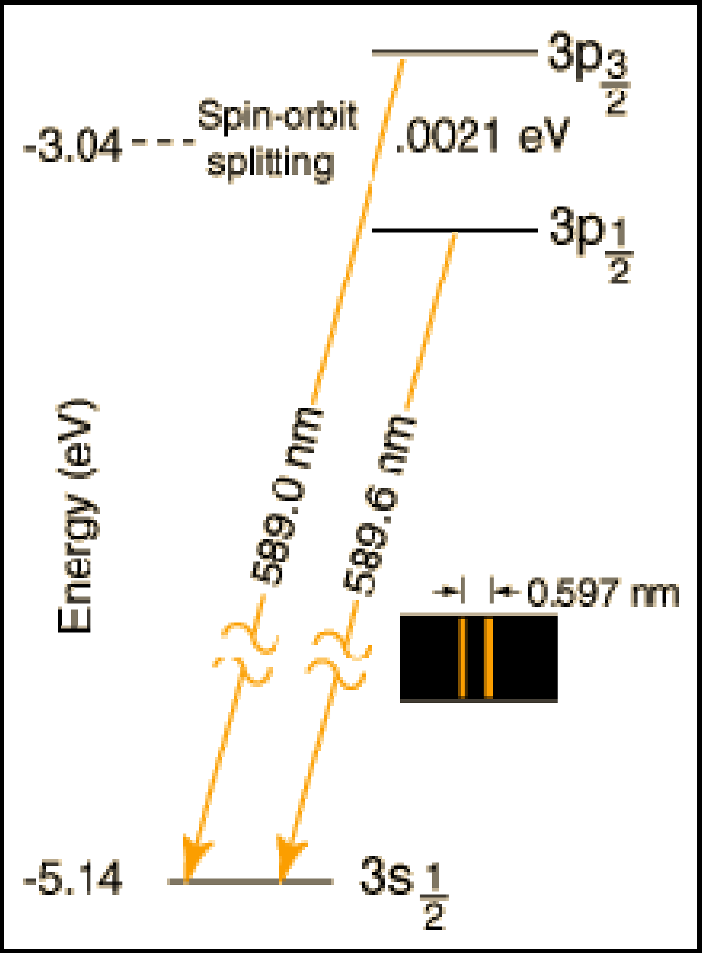


Modern Uses:

- Nuclear magnetic Resonance Spectroscopy
- Electron-spin resonance spectroscopy
- Magnetic Resonance Imaging
- Mössbauer spectroscopy

Sodium Doublet

- A sodium Yellow doublet transition happens because of transition from 3p to 3s transition.
- The 3p level is split into states with total angular momentum (j=l+s) of j=3/2 and j=1/2 by the magnetic energy of the electron spin in the presence of the internal magnetic field caused by the orbital motion.
- In the case of the sodium doublet, the difference in energy for the $3p_{3/2}$ and $3p_{1/2}$ comes from a change of 1 unit in the spin orientation with the orbital part presumed to be the same.



Sodium Doublet Continued

Energy Difference

For Sodium doublet, $\Delta j = 1$ as there is only spin shift from 1/2 to -1/2 or vice-versa (m remains same)

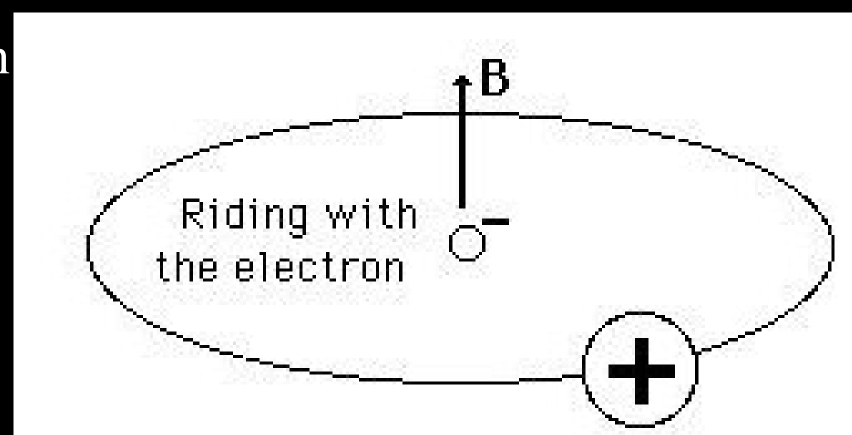
g = Gyromagnetic ratio = 2.002319304386

B = magnetic field produced by the Nucleus when observed from the electron reference frame (Classical View point)

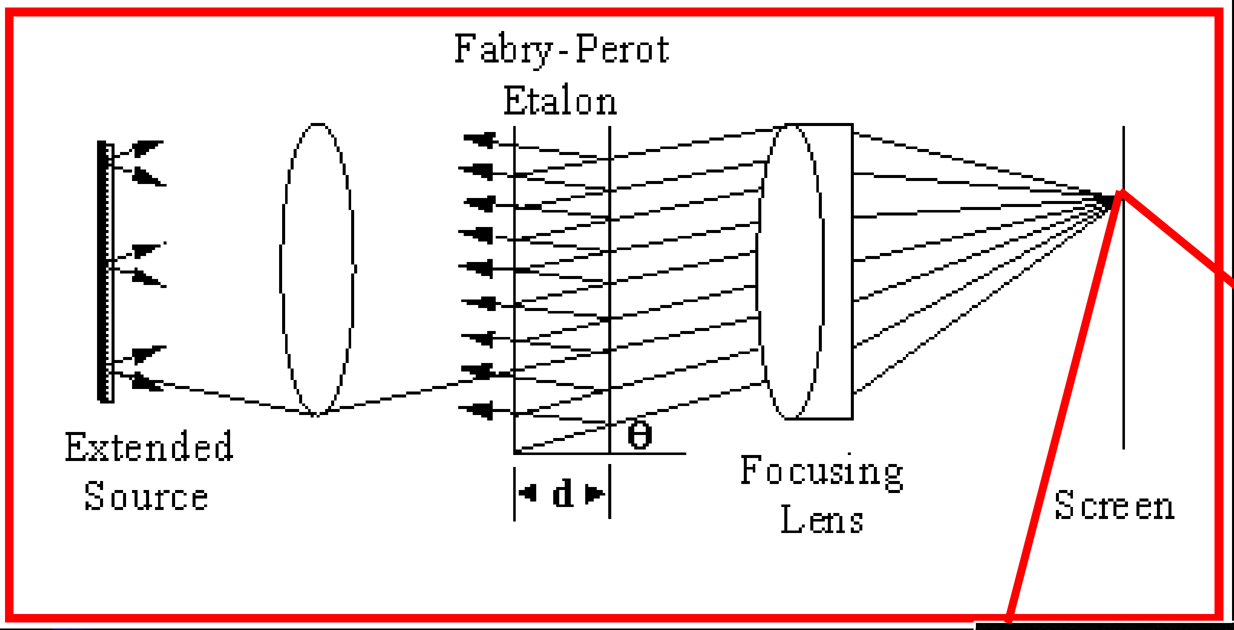
$$\Delta E = \Delta j \cdot \left(\frac{e}{2m_{.e}} \right) \cdot g \cdot B$$

$$\Delta E_{.th} = 0.0021 eV$$

$$B_{.th} = 18T$$



The Fabry-Perot Interferometer



Path Difference for bright fringe:

$$\Delta = 2n_f d(\cos(\theta)) = m\lambda$$

Fringes result from interference between multiple reflected beams, with bright fringes corresponding to constructive interference and dark fringes to destructive interference

Theory

2 Fringe Patterns from Wavelength Components of light source coincide when:

$$\Delta OPL = m_1\lambda_1 = m_2\lambda_2$$

Next Coincidence occurs at:

$$\Delta OPL = (m_1 + n)\lambda_1 = (m_2 + n + 1)\lambda_2$$

Where n = number of fringes between coincidences

$$\Delta\lambda = \lambda_1 - \lambda_2 = \frac{\lambda_2}{n}$$

Theory

Number of fringes is can be related to the mirror displacement by:

$$n = \frac{2d}{\lambda_1}$$

Therefore, the difference in wavelengths is approximately:

$$\Delta\lambda \approx \frac{\lambda_1^2}{2d} \approx \frac{\lambda_2^2}{2d} \qquad (1)$$

However, using average wavelength gives a more accurate result:

$$\Delta\lambda = \frac{\lambda_{avg}^2}{2d} \qquad (2)$$

Since the average requires both wavelengths to be known, with only 1 wavelength, equation 1 can be used to get a first approximation. The results can then be used with equation 2 to get a more accurate value.

Theory Question

- For reflectivity of R=0.85,

Coefficient of Finesse:

$$F = \frac{4R}{(1-R)^2} = \frac{4*0.85}{(1-0.85)^2} = 151.11$$

Finesse: $\mathcal{F}$

$$= \frac{\pi(F)^{1/2}}{2} = 19.31$$

Minimum Wavelength Increment:

$$(\Delta\lambda_0)_{\min} = \frac{\lambda_0^2}{Finesse*2n_f d} = \frac{(588.995nm)^2}{19.31*2*1*159149nm} = 0.0564nm$$

Free Spectral Range:

$$(\lambda_0)_{FSR} = Finesse*(\lambda_0)_{\min} = 19.31*0.00564nm$$

$$(\lambda_0)_{FSR} = 1.089nm$$

Resolving Power: $\mathcal{R}$

$$= \frac{\lambda_0}{(\Delta\lambda_0)_{\min}} = \frac{588.995nm}{0.0564nm} = 1.04*10^4\,nm$$

Theory Question

Minimum Frequency Increment:

$$(\Delta v_0)_{\min} = \frac{c}{(\Delta\lambda_0)\min} = \frac{3*10^8 m/s}{0.0564*10^{-9} m}$$

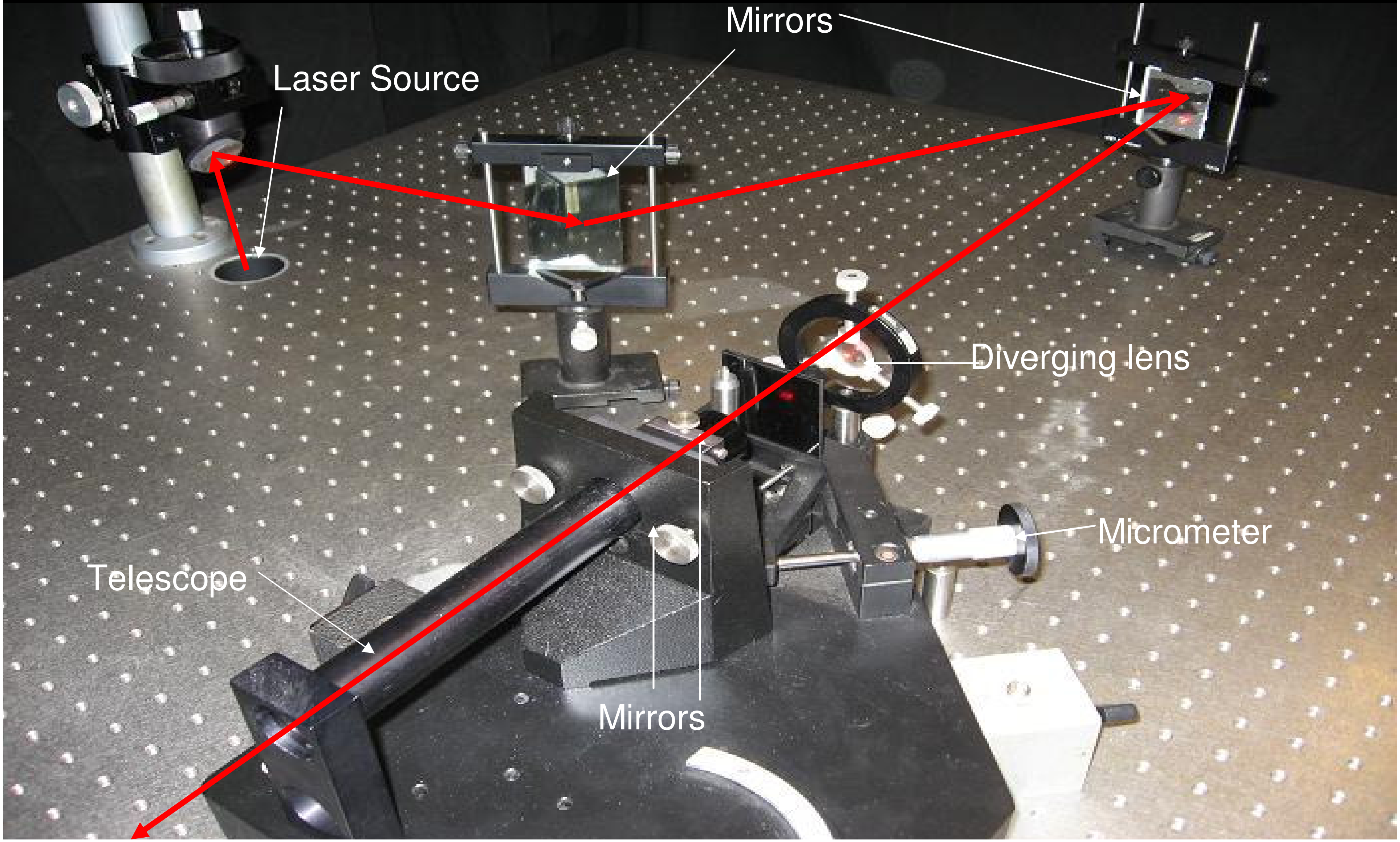
$$(\Delta v_0)_{\min} = 5.319 Hz$$

Free Spectral Frequency Range:

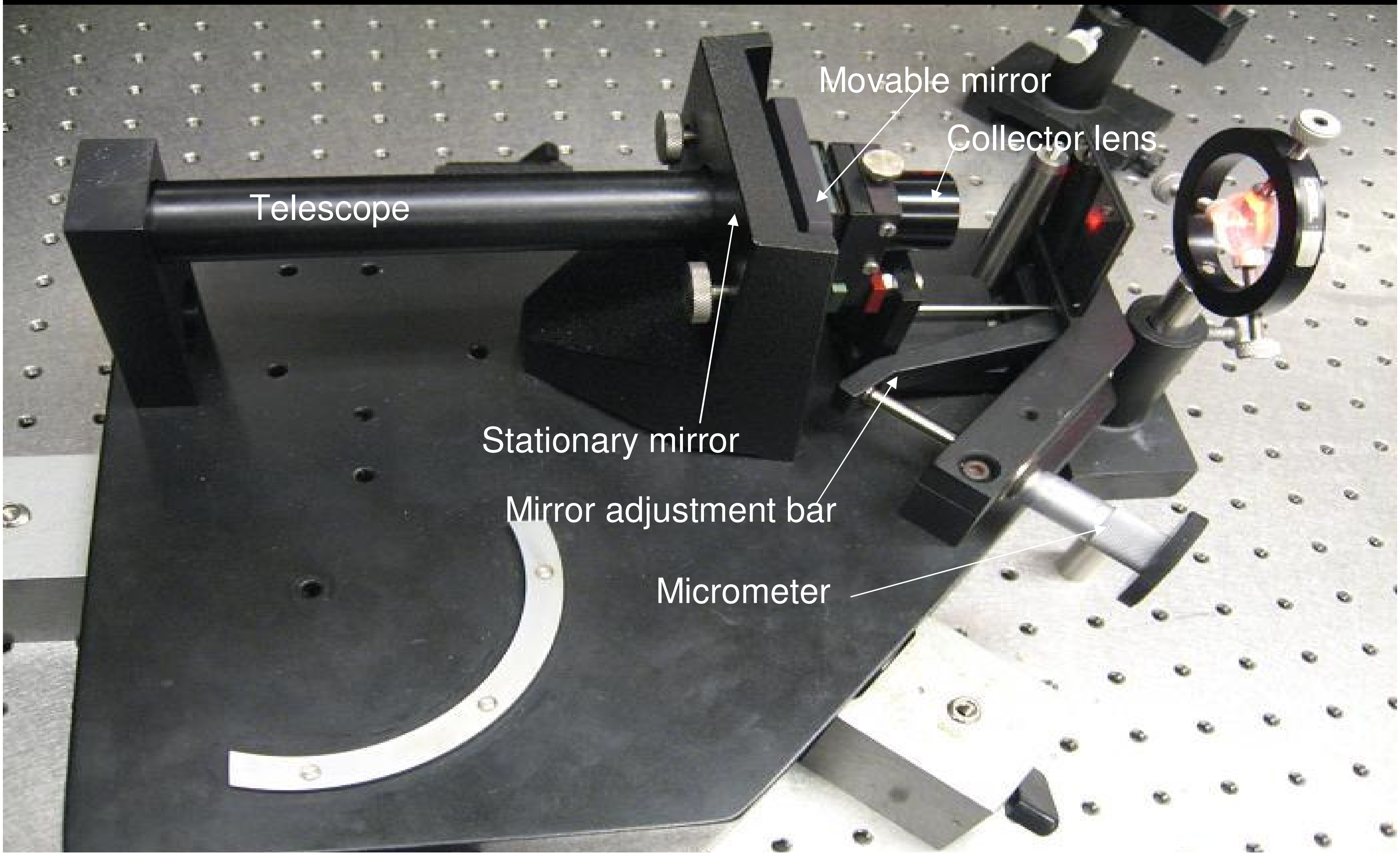
$$(v_0)_{FSR} = \frac{c}{(\lambda_0)_{FSR}} = \frac{3*10^8 m/s}{1.089*10^{-9} m}$$

$$(\Delta v_0)_{\min} = 2.755*10^{17} Hz$$

Experimental Setup



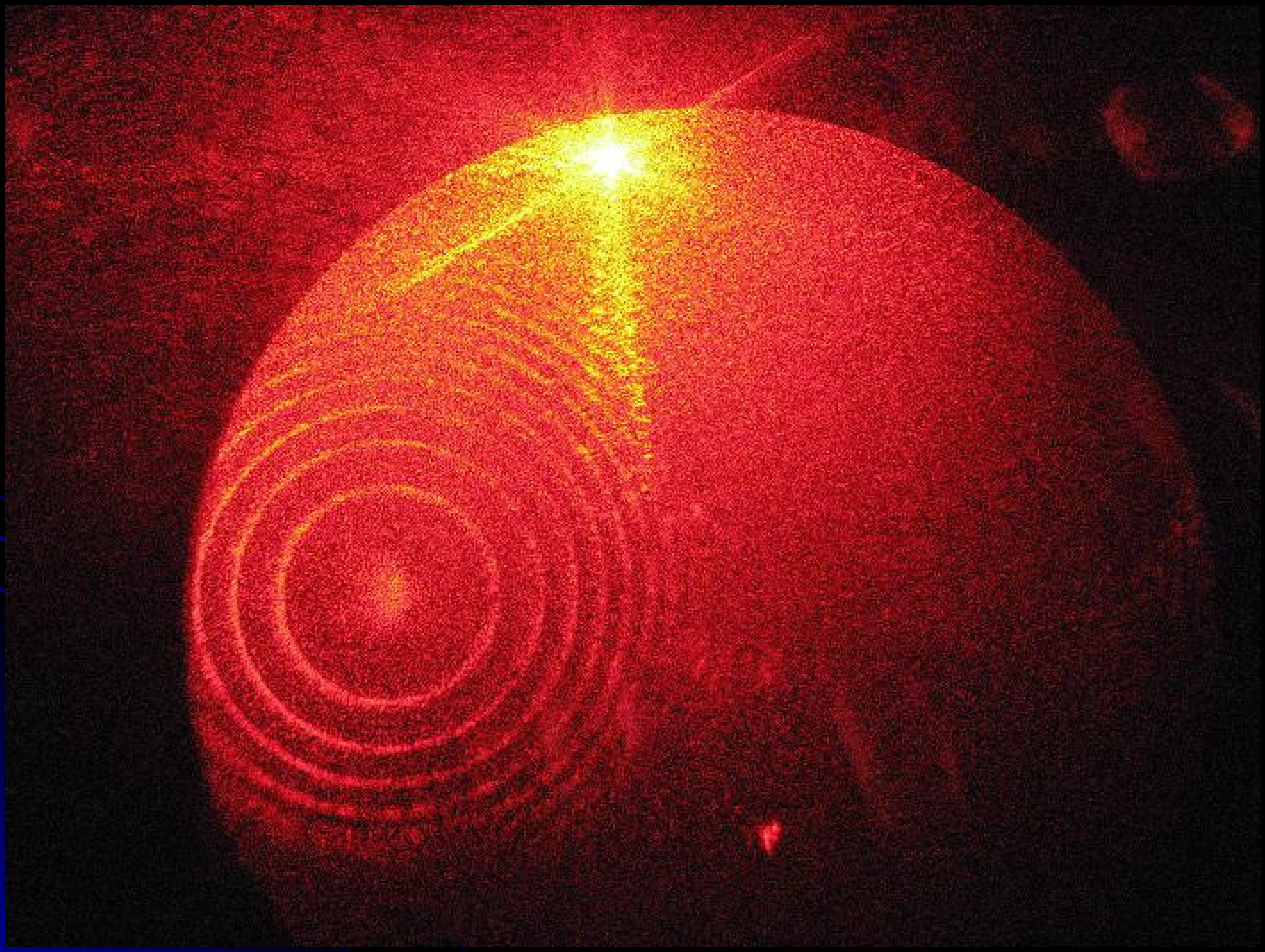
Experimental Setup



Procedure – Calibration

- Assemble Fabry-Perot interferometer with the mirrors close, but not touching
- Record the value on the micrometer
- Rotate micrometer and count the number of fringes that pass
- Record the micrometer reading every 50 fringes until 500 fringes have passed

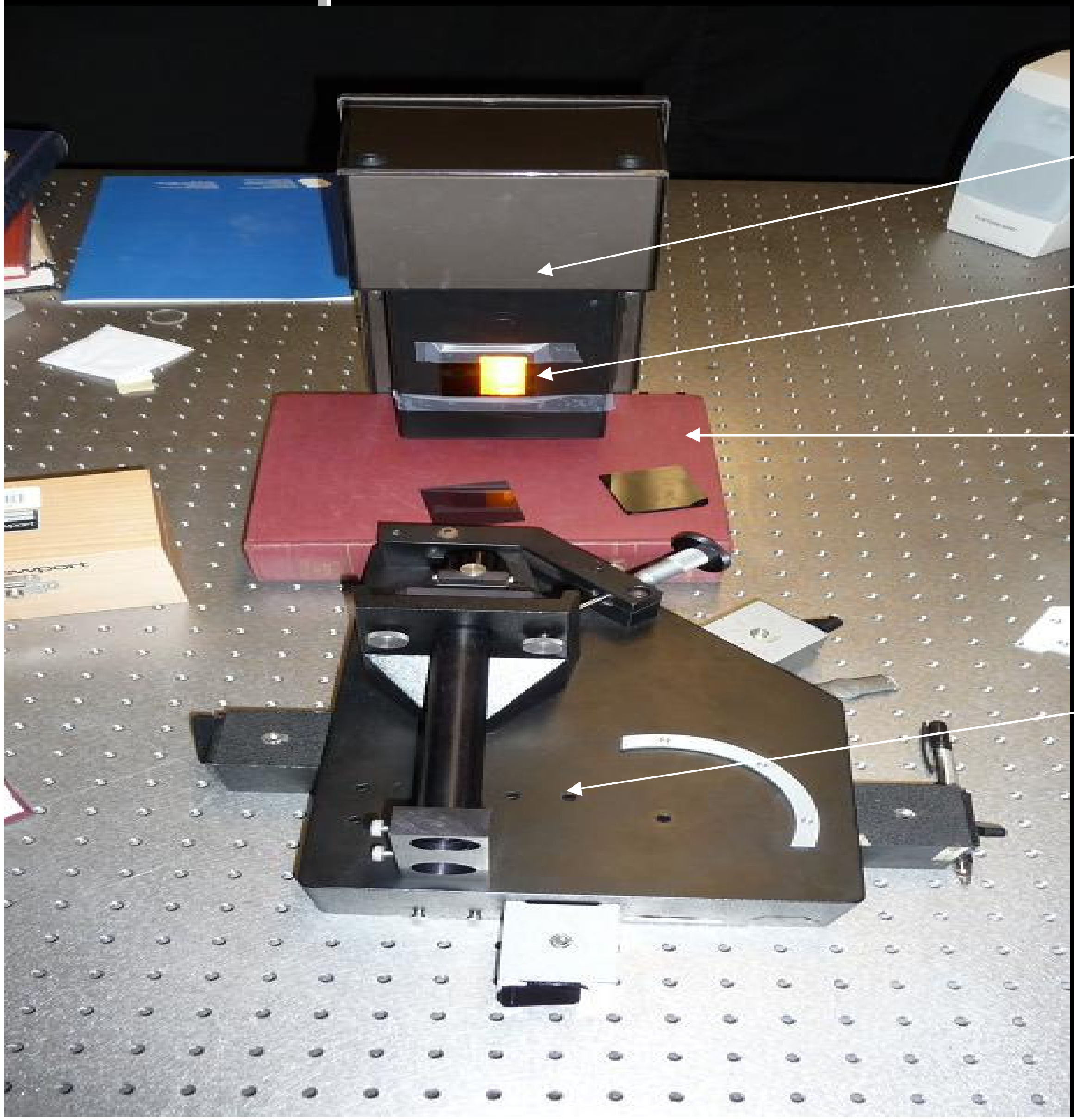
Procedure – Calibration



Procedure – Calibration

- Convert fringe count to mirror displacement and plot mirror displacement vs. micrometer displacement
- Slope of graph is the calibration factor, the amount the mirror moves per tic on the micrometer

Experimental Setup - Sodium



Sodium lamp

Filter

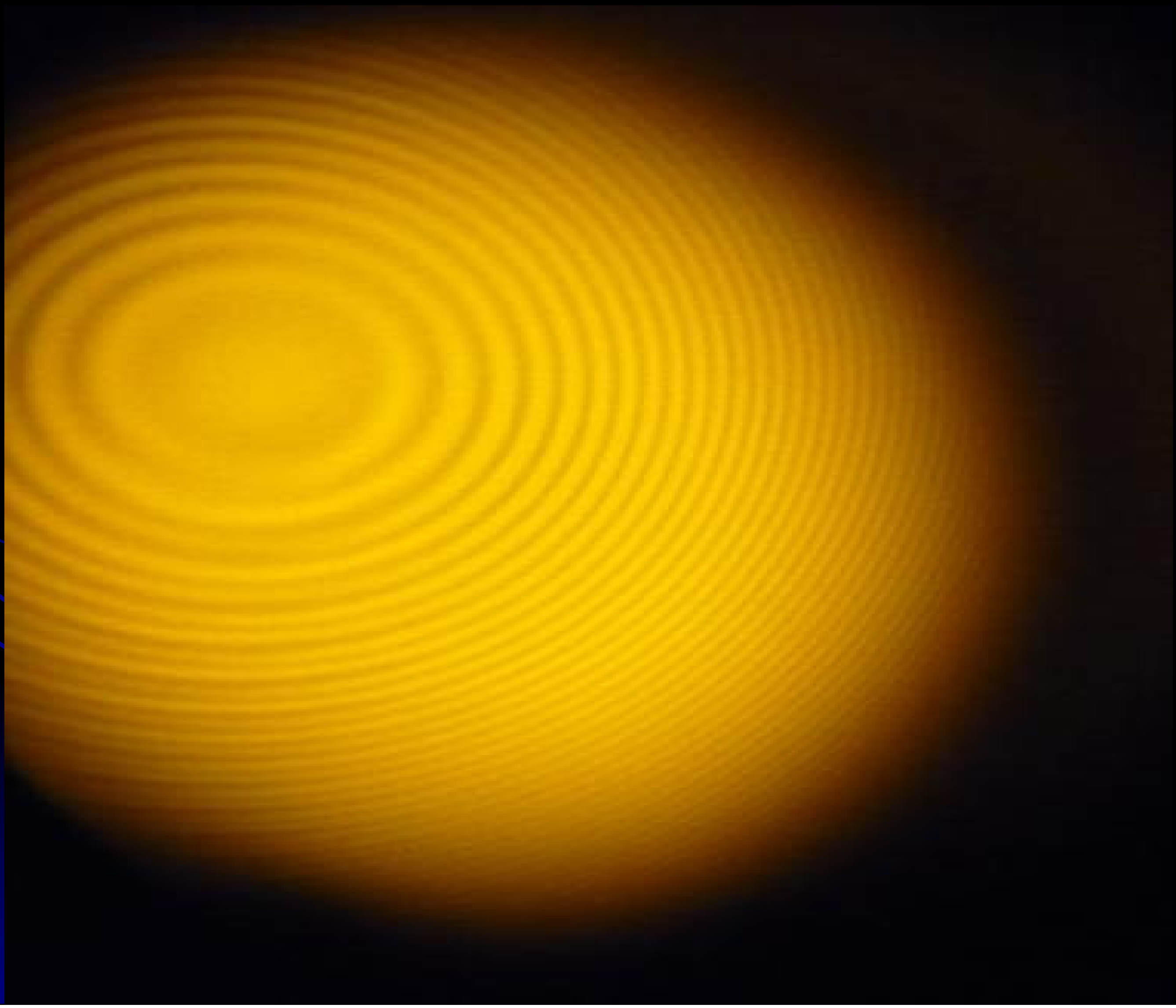
Precision leveling device
(Handbook of
Mathematical Functions)

Interferometer
(same setup as laser)

Procedure – Sodium Doublet

- Replace laser source with sodium source
- Sodium produces 2 sets of fringe patterns
- Adjust mirror separation until 2 patterns are coincident (each fringe is made up of 2 closely spaced lines)
- Rotate micrometer in both directions and record position where the 2 close lines start to blur, average the two numbers to determine the position of coincidence
- Rotate micrometer further and find another position of coincidence
- Repeat for 6+ coincidences

Procedure – Sodium Doublet



Procedure – Sodium Doublet

Procedure – Sodium Doublet

Procedure – Sodium Doublet

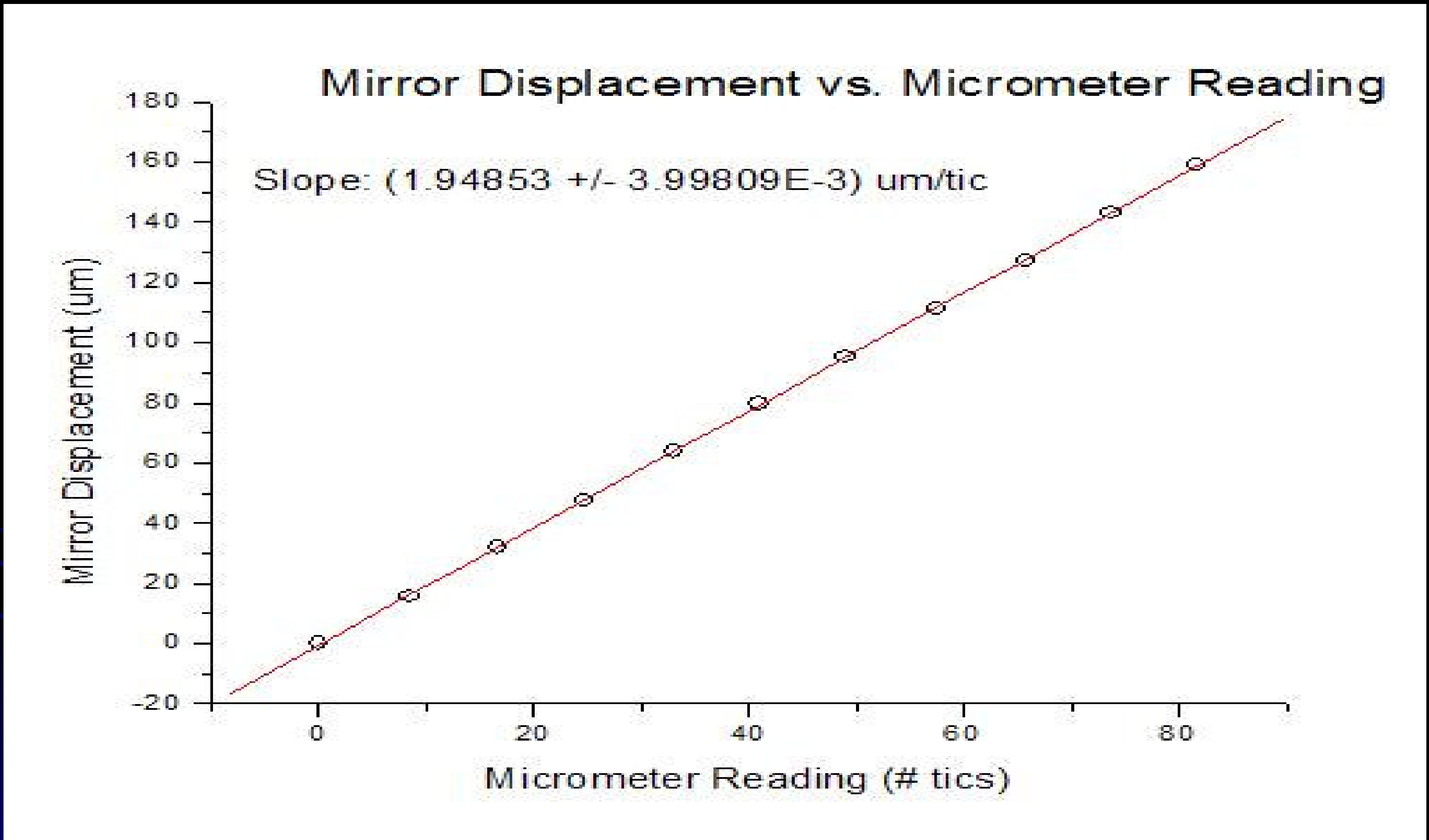
- Convert micrometer displacements to mirror displacements using previous calibration factor
- Plot mirror displacement vs. coincidence number
- Slope is d, the distance the mirror must move between two coincidences
- Calculate the energy difference between the two 3p states
- Using that, calculate the effective nuclear charge and the magnetic field

Results - Calibration

	CircularReading(X)	FringeCount	Wavelength	MirrorDisplacement(Y)
1	0	0	6.328E-7	0
2	8.5	50	6.328E-7	1.582E-5
3	16.75	101	6.328E-7	3.19564E-5
4	24.75	151	6.328E-7	4.77764E-5
5	33	202	6.328E-7	6.39128E-5
6	41	252	6.328E-7	7.97328E-5
7	49	302	6.328E-7	9.55528E-5
8	57.5	352	6.328E-7	1.11373E-4
9	65.75	402	6.328E-7	1.27193E-4
10	73.75	453	6.328E-7	1.43329E-4
11	81.75	503	6.328E-7	1.59149E-4

$$MirrorDisp\ lacement = \frac{FringeCoun\ t * Wavelength}{2}$$

Results - Calibration



$$Calibratio\ nFactor = (1.94853 \pm 3.99809\,E-3)\,\mu m$$

Results – Sodium Double (Trial 1)

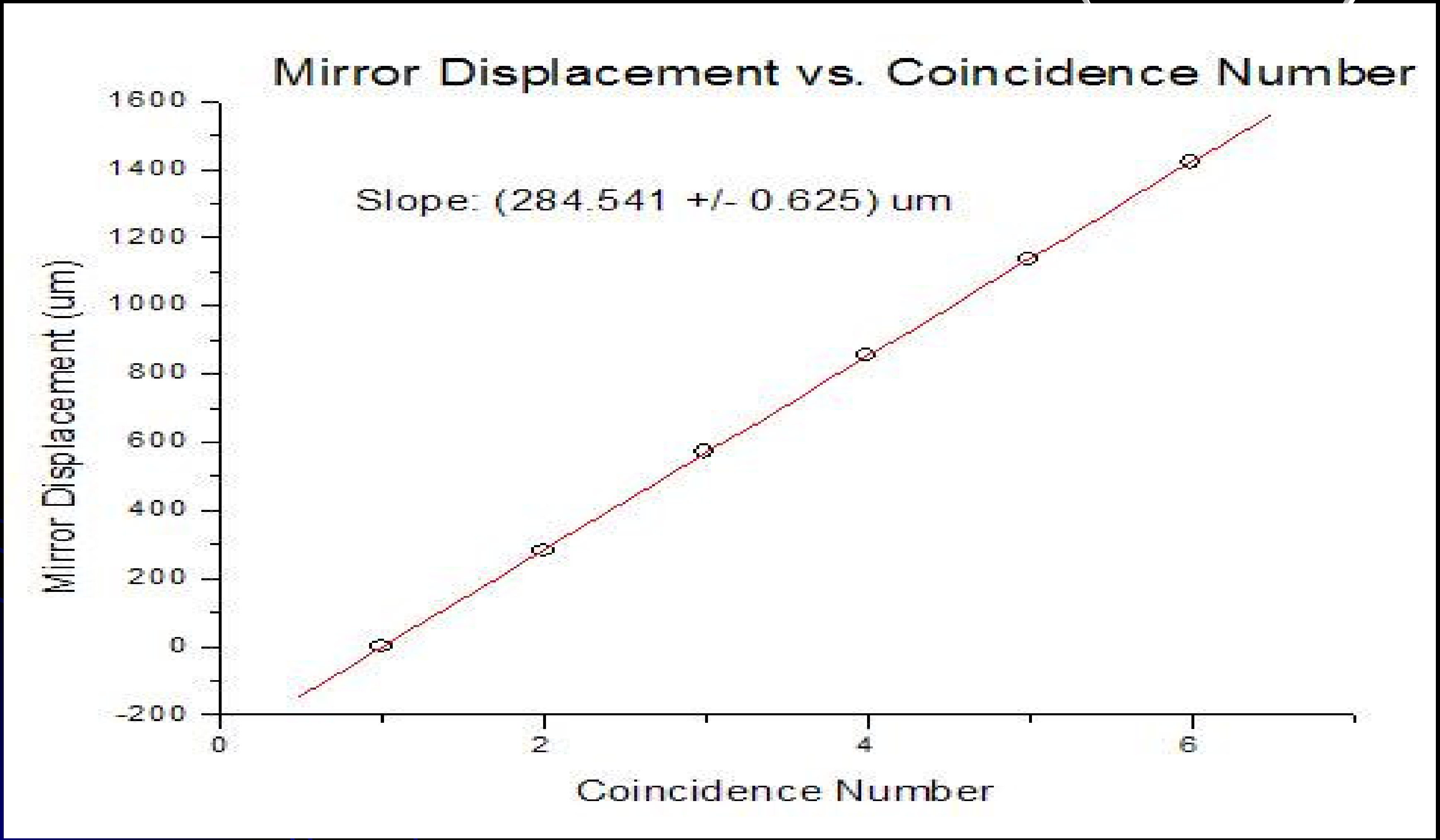
	CoincidenceNum(X)	CircularReading1	CircularReading2	AvgCircReading	CircDisplacement	MirrorDisplace(Y)
1	6	1299	1277.5	1288.25	729.5	0.001421
2	5	1148.5	1135	1141.75	583	0.001136
3	4	1005	990	997.5	438.75	8.549175E-4
4	3	859.5	844	851.75	293	5.709193E-4
5	2	708.5	696.5	702.5	143.75	2.801012E-4
6	1	567.5	550	558.75	0	0

$$AvgCirc\,\mathrm{Re}\,ading = \frac{Circular\,\mathrm{Re}\,ading1 + Circular\,\mathrm{Re}\,ading\,2}{2}$$

$$CircDisplacement = AvgCirc\,\mathrm{Re}\,ading - 558.75$$

$$MirrorDisp\ lace = CircDispla\ cement * Calibratio\ nFactor$$

Results – Sodium Doublet (Trial 1)



$$d = (284.541 \pm 0.625)\mu m$$

Results – Sodium Doublet (Trial 1)

$$\lambda_{1theo} := 589.592\,nm$$

$$\Delta\lambda_1 := \frac{\lambda_{1theo}^{\;2}}{2\cdot d} = 0.611\cdot nm$$

$$\Delta\lambda := \left[\begin{array}{l} \Delta\lambda \leftarrow \Delta\lambda_1 \\ \text{for } i \in 1..100 \\ \quad \lambda_2 \leftarrow \lambda_{1theo} - \Delta\lambda \\ \quad \lambda_{avg} \leftarrow \dfrac{(\lambda_{1theo} + \lambda_2)}{2} \\ \quad \Delta\lambda \leftarrow \dfrac{\lambda_{avg}^{\;2}}{2\cdot d} \end{array}\right]$$

$$\Delta\lambda = 0.6102088333000636\,nm$$

$$\Delta\lambda_{theo} := 0.597nm$$

$$\%err_{\Delta\lambda} := \frac{(\Delta\lambda - \Delta\lambda_{theo})\cdot 100}{\Delta\lambda_{theo}} = 2.213$$

$$\lambda_2 := \lambda_{1theo} - \Delta\lambda = 588.982nm$$

$$\lambda_{2theo} := 588.995\,nm$$

$$\%err_{\lambda 2} := \frac{(\lambda_2 - \lambda_{2theo})\cdot 100}{\lambda_{2theo}} = -2.243 \times 10^{-3}$$

Results – Sodium Doublet (Trial 1)

Energy separation between states:

$$\Delta E := \frac{(h \cdot c \cdot \Delta\lambda)}{{\lambda_{avg}}^2} = 3.4905756734853055 \times 10^{-22} J$$

$$\Delta E_{ev} := \frac{\Delta E}{1.602 \cdot 10^{-19} J} = 2.179 \times 10^{-3}$$

$$\Delta E_{evtheo} := .0021$$

$$\%err := \frac{(\Delta E_{ev} - \Delta E_{evtheo}) \cdot 100}{\Delta E_{evtheo}} = 3.756$$

Internal magnetic field:

$$m_s := \frac{1}{2} \qquad e := 1.60217646 \cdot 10^{-19} C$$

$$h_b := 6.595 \cdot 10^{-16} s \qquad m_e := 9.10938188 \cdot 10^{-31} kg$$

$$B := \left(\frac{\Delta E_{ev}}{2 \cdot h_b \cdot m_s \cdot e} \right) \cdot m_e = 18.784T$$

$$\%err_B := \frac{(B - 18T) \cdot 100}{18T} = 4.358$$

Effective nuclear charge:

$$n := 3 \qquad l := 1$$

$$Z_{eff} := \sqrt{\frac{\Delta E_{ev} \cdot n^3 \cdot l \cdot (l + 1)}{7.24 \cdot 10^{-4}}} = 12.748$$

Results – Sodium Double (Trial 2)

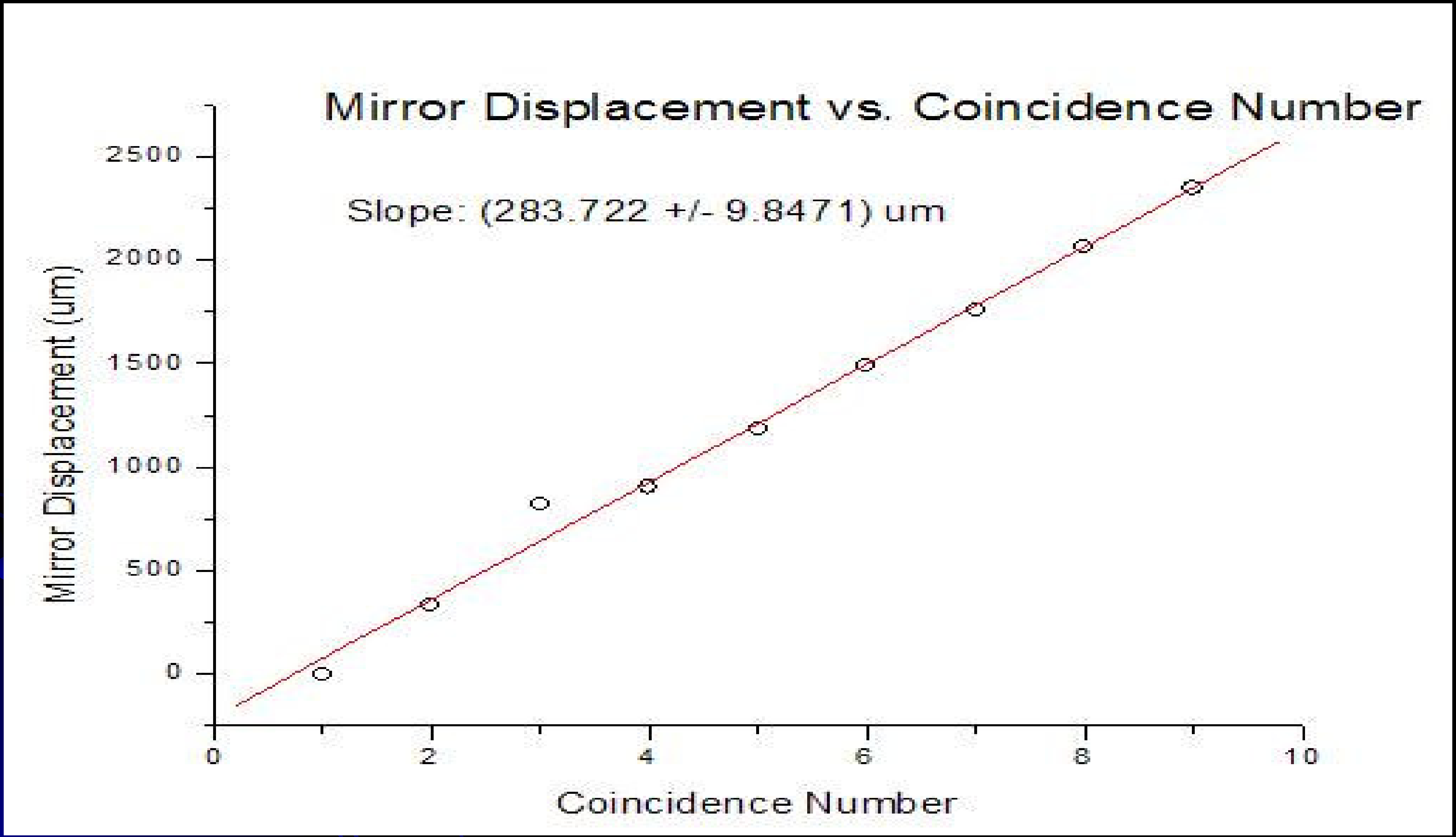
	CoincidenceNum(X)	CircularReading1	CircularReading2	AvgCircReading	CircDisplacement	MirrorDisplace(Y)
1	9	1298	1289.5	1293.75	1204.25	0.00235
2	8	–	–	1147	1057.5	0.00206
3	7	1005	980	992.5	903	0.00176
4	6	859	848	853.5	764	0.00149
5	5	705	687.5	696.25	606.75	0.00118
6	4	558	548.5	553.25	463.75	9.03631E-4
7	3	515	509.5	512.25	422.75	8.23741E-4
8	2	267.5	254	260.75	171.25	3.33686E-4
9	1	91	88	89.5	0	0

$$AvgCircReading = \frac{CircularReading1 + CircularReading2}{2}$$

$$CircDisplacement = AvgCircReading - 558.75$$

$$MirrorDisplace = CircDisplacement * CalibrationFactor$$

Results – Sodium Doublet (Trial 2)



$$d = (283.722 \pm 9.8471)\mu m$$

Results – Sodium Doublet (Trial 2)

$$\lambda_{1theo} := 589.592\,nm$$

$$\Delta\lambda_1 := \frac{\lambda_{1theo}^2}{2\cdot d} = 0.613\,nm$$

$$\Delta\lambda := \begin{array}{l} \Delta\lambda \leftarrow \Delta\lambda_1 \\ \text{for } i \in 1..100 \\ \quad \lambda_2 \leftarrow \lambda_{1theo} - \Delta\lambda \\ \quad \lambda_{avg} \leftarrow \dfrac{(\lambda_{1theo} + \lambda_2)}{2} \\ \quad \Delta\lambda \leftarrow \dfrac{\lambda_{avg}^2}{2\cdot d} \end{array}$$

$$\Delta\lambda = 0.6119681211422627\cdot nm$$

$$\Delta\lambda_{theo} := 0.597\,nm$$

$$\lambda_2 := \lambda_{1theo} - \Delta\lambda = 588.98\,nm$$

$$\%err_{\Delta\lambda} := \frac{(\Delta\lambda - \Delta\lambda_{theo})\cdot 100}{\Delta\lambda_{theo}} = 2.507$$

$$\%err_{\lambda 2} := \frac{(\lambda_2 - \lambda_{2theo})\cdot 100}{\lambda_{2theo}} = -2.541\times 10^{-3}$$

Results – Sodium Doublet (Trial 2)

Energy separation between states:

$$\Delta E := \frac{(h \cdot c \cdot \Delta\lambda)}{\lambda_{avg}^{2}} = 3.500649773220736 \times 10^{-22} J$$

$$\Delta E_{ev} := \frac{\Delta E}{1.602 \cdot 10^{-19} J} = 2.185 \times 10^{-3}$$

$$\Delta E_{evtheo} := .0021$$

$$\%err := \frac{\left(\Delta E_{ev} - \Delta E_{evtheo}\right) \cdot 100}{\Delta E_{evtheo}} = 4.056$$

Internal magnetic field:

$$m_s := \frac{1}{2} \qquad e := 1.60217646 \cdot 10^{-19} C$$

$$h_b := 6.595 \cdot 10^{-16} s \qquad m_e := 9.10938188 \cdot 10^{-31} kg$$

$$B := \left(\frac{\Delta E_{ev}}{2 \cdot h_b \cdot m_s \cdot e}\right) \cdot m_e = 18.839\ T$$

$$\%err_B := \frac{(B - 18T) \cdot 100}{18T} = 4.659$$

Effective nuclear charge:

$$n := 3 \qquad l := 1$$

$$Z_{eff} := \sqrt{\frac{\Delta E_{ev} \cdot n^{3} \cdot l \cdot (l + 1)}{7.24 \cdot 10^{-4}}} = 12.766$$

Conclusions

- Wavelength separation

$$\Delta\lambda = 0.6102088333000636\,\text{nm}$$

$$\%err_{\Delta\lambda} := \frac{(\Delta\lambda - \Delta\lambda_{theo})\cdot 100}{\Delta\lambda_{theo}} = 2.213$$

$$\Delta\lambda = 0.6119681211422627\cdot\text{nm}$$

$$\%err_{\Delta\lambda} := \frac{(\Delta\lambda - \Delta\lambda_{theo})\cdot 100}{\Delta\lambda_{theo}} = 2.507$$

- Energy difference

$$\Delta E := \frac{(h\cdot c\cdot\Delta\lambda)}{{\lambda_{avg}}^{2}} = 3.4905756734853055\times 10^{-22}\,\text{J}$$

$$\Delta E_{ev} := \frac{\Delta E}{1.602\cdot 10^{-19}\,\text{J}} = 2.179\times 10^{-3}$$

$$\%err := \frac{(\Delta E_{ev} - \Delta E_{evtheo})\cdot 100}{\Delta E_{evtheo}} = 3.756$$

$$\Delta E := \frac{(h\cdot c\cdot\Delta\lambda)}{{\lambda_{avg}}^{2}} = 3.5006497732207364\times 10^{-22}\,\text{J}$$

$$\Delta E_{ev} := \frac{\Delta E}{1.602\cdot 10^{-19}\,\text{J}} = 2.185\times 10^{-3}$$

$$\%err := \frac{(\Delta E_{ev} - \Delta E_{evtheo})\cdot 100}{\Delta E_{evtheo}} = 4.056$$

Conclusions

- Effective nuclear charge

$$Z_{eff} := \sqrt{\frac{\Delta E_{ev} \cdot n^3 \cdot l \cdot (l + 1)}{7.24\,10^{-4}}} = 12.748$$

$$Z_{eff} := \sqrt{\frac{\Delta E_{ev} \cdot n^3 \cdot l \cdot (l + 1)}{7.24 \cdot 10^{-4}}} = 12.766$$

- Magnetic field

$$B := \left(\frac{\Delta E_{ev}}{2 \cdot h_b \cdot m_s \cdot e}\right) \cdot m_e = 18.784T$$

$$\%err_B := \frac{(B - 18T) \cdot 100}{18T} = 4.358$$

$$B := \left(\frac{\Delta E_{ev}}{2 \cdot h_b \cdot m_s \cdot e}\right) \cdot m_e = 18.839\ T$$

$$\%err_B := \frac{(B - 18T) \cdot 100}{18T} = 4.659$$

Conclusions/Observations

- Interferometer is extremely difficult to align and keep aligned, slightest bump throws the whole thing off, had to re-align using laser
- When rotating micrometer to increase mirror separation, spring pulling mirror back did not always do so smoothly or had to be manually pushed back
- Fringes became narrower as mirror separation decreased, made it very hard to see if there was coincidence or not
- Results ended up being very accurate despite problems, because once it was aligned and your eyes were used to seeing the fringes, they were extremely clear and could easily be read

Sources of Error

- Eyestrain
  - Too much staring at fringes in one day and they start to be harder to focus on
  - Random
- Measurement limitations
  - Micrometer precision was limited
  - Random
- Minor misalignment
  - Mirrors might not have been perfectly parallel, could throw readings off slightly
  - Random
- Resolution of device
  - Though we used the range of positions where we could separate the two closely spaced lines as our coincidence range, we could not resolve the fringes enough to be able to identify the exact position of maximum coincidence
  - Narrow fringes at smaller mirror separation made position of coincidence even harder to see
  - Systematic – device limitations
  - Random – human vision limitations

References

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- Lecture notes
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