

Thickness of a Thin Film

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Max Orseno

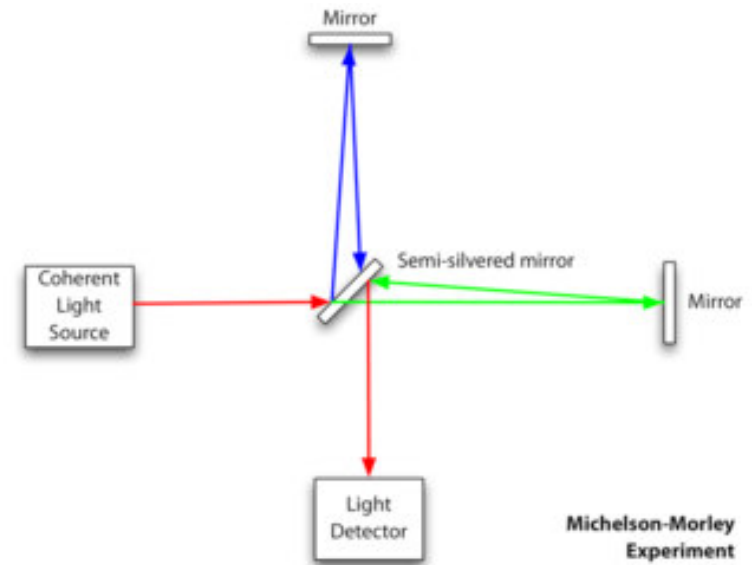
Achille Nicoletti

6 o'clock p.m., March 10, 2010

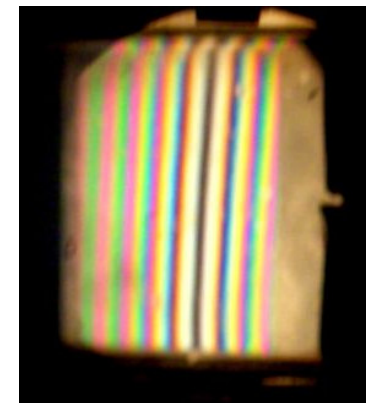
Optics Laboratory, PHY455

Theory: Michelson Interferometer

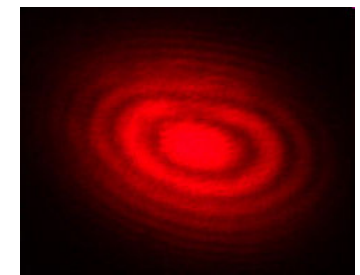
- Invented by Albert Michelson in 1881
- Amplitude splitting device
 - Incident beam is split into reflected and transmitted beams, each with a smaller amplitude than incident
- Interference of split/recombined beams produces circular or straight-line fringes
 - Fringe characteristics directly related to difference in OPD of two beams
- Interpretation of fringes allows for:
 - Comparison of wavelengths
 - Measurement of refractive index of gases and transparent solids.



http://en.wikibooks.org/wiki/Special_Relativity/Print_version



<http://www.millersville.edu/physics/experiments/current.php>



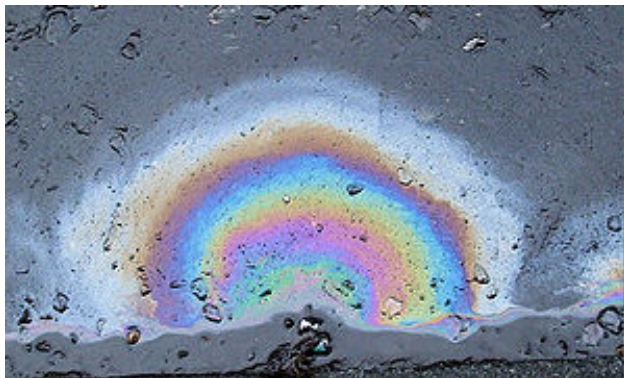
<http://www.search.com/reference/Kilogram>

Theory: Thin Film Interference

- Start at the beginning: interference effects are observable in sheet transparent materials
 - Thin film: a layer of sheet transparent material whose thickness is of the order of a given wavelength of electromagnetic radiation



<http://twilit.wordpress.com/2008/03/15/bubbles-and-interference/>



<http://www.answers.com/topic/optics>



<http://electron9.phys.utk.edu/phys136d/modules/m9/film.htm>

Theory: OPD

OPD = Optical Path Difference

$$\Lambda = n_f \cdot [(AB) + (BC)] - n_1 \cdot (AD) \longrightarrow (AB) = (BC) = \frac{d}{\cos(\theta_t)}$$

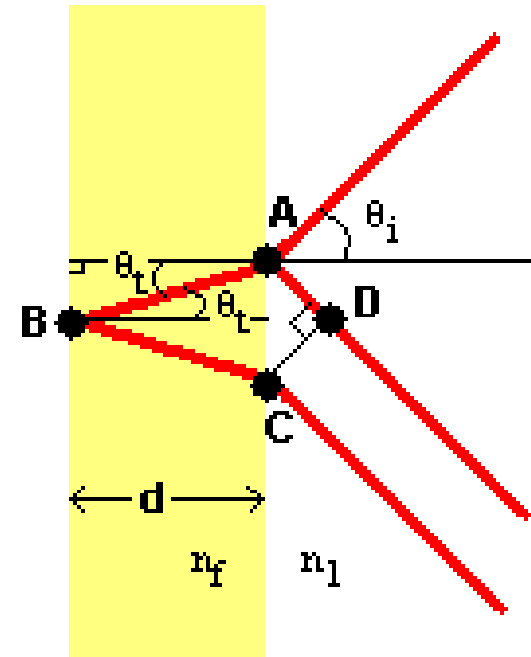
$$AD = (AC) \cdot \sin(\theta_i) \longleftarrow \Lambda = \frac{2 \cdot n_f \cdot d}{\cos(\theta_t)} - n_1 \cdot (AD)$$

Snell's Law:

$$(AD) = (AC) \cdot \frac{n_f}{n_1} \cdot \sin(\theta_t)$$

$$(AC) = 2 \cdot \tan(\theta_t)$$

$$\Lambda = \frac{2 \cdot n_f \cdot d}{\cos(\theta_t)} \cdot (1 - \sin(\theta_t)^2) \longrightarrow \Lambda = 2 \cdot n_f \cdot d \cdot \cos(\theta_t)$$



$$\text{OPD} = \Lambda = 2 \cdot n_f \cdot d \cdot \cos(\theta_t)$$

Theory: Min/Max Conditions

$$\text{Phase Difference} = \delta \quad \text{OPD} = \Lambda = 2 \cdot n_f \cdot d \cdot \cos(\theta_t) \quad k_0 = \frac{2 \cdot \pi}{\lambda_0}$$

$$\delta = k_0 \cdot \Lambda \pm \pi \quad \delta = \frac{4 \cdot \pi \cdot n_f}{\lambda_0} \cdot d \cdot \cos(\theta_t) \pm \pi$$

$$\delta = \frac{4 \cdot \pi \cdot d}{\lambda_0} \cdot \left(n_f^2 - n^2 \cdot \sin^2(\theta_i) \right)^{\frac{1}{2}} \cdot \pm \pi$$

Maximum at $\delta = 2 \cdot m \cdot \pi$

$$2 \cdot m \cdot \pi = \frac{4 \cdot \pi \cdot n_f}{\lambda_0} \cdot d \cdot \cos(\theta_t) - \pi$$

$$\pi(2 \cdot m + 1) = \frac{4 \cdot \pi \cdot n_f}{\lambda_0} \cdot d \cdot \cos(\theta_t) \quad \left(\frac{n_f}{\lambda_0} = \lambda_f \right)$$

$$d \cdot \cos(\theta_t) = (2 \cdot m + 1) \cdot \frac{\lambda_f}{4} \quad (\text{maxima})$$

Minimum at $\delta = (2 \cdot m \pm 1) \cdot \pi$

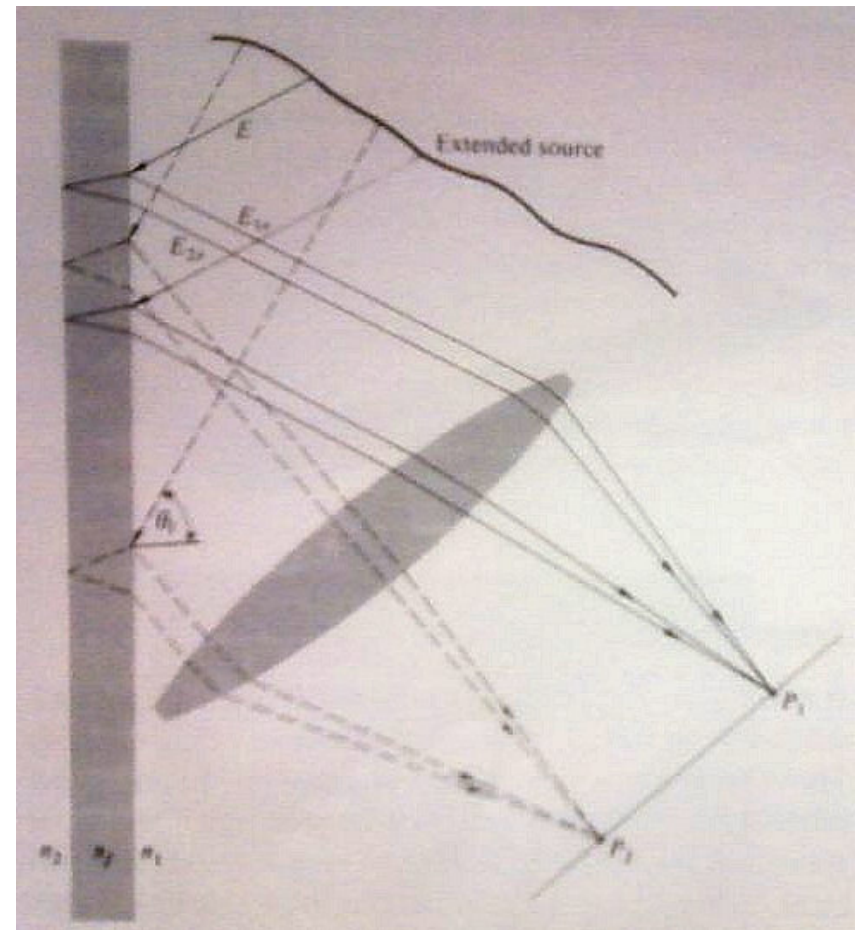
$$(2 \cdot m - 1) \cdot \pi = \frac{4 \cdot \pi \cdot n_f}{\lambda_0} \cdot d \cdot \cos(\theta_t) - \pi$$

$$2 \cdot m \cdot \pi = \frac{4 \cdot \pi \cdot n_f}{\lambda_0} \cdot d \cdot \cos(\theta_t) \quad \left(\frac{n_f}{\lambda_0} = \lambda_f \right)$$

$$d \cdot \cos(\theta_t) = m \cdot \frac{\lambda_f}{2} \quad (\text{minima})$$

Theory: Fringes of Equal Inclination

- Fringes of Equal Inclination
 - Formed by different rays of incident light ALL WITH SAME INCIDENT ANGLE
- If source is extended, there will be many rays of different incident angles—each will form a fringe
 - Rays with larger OPD between them form fringes closest to center



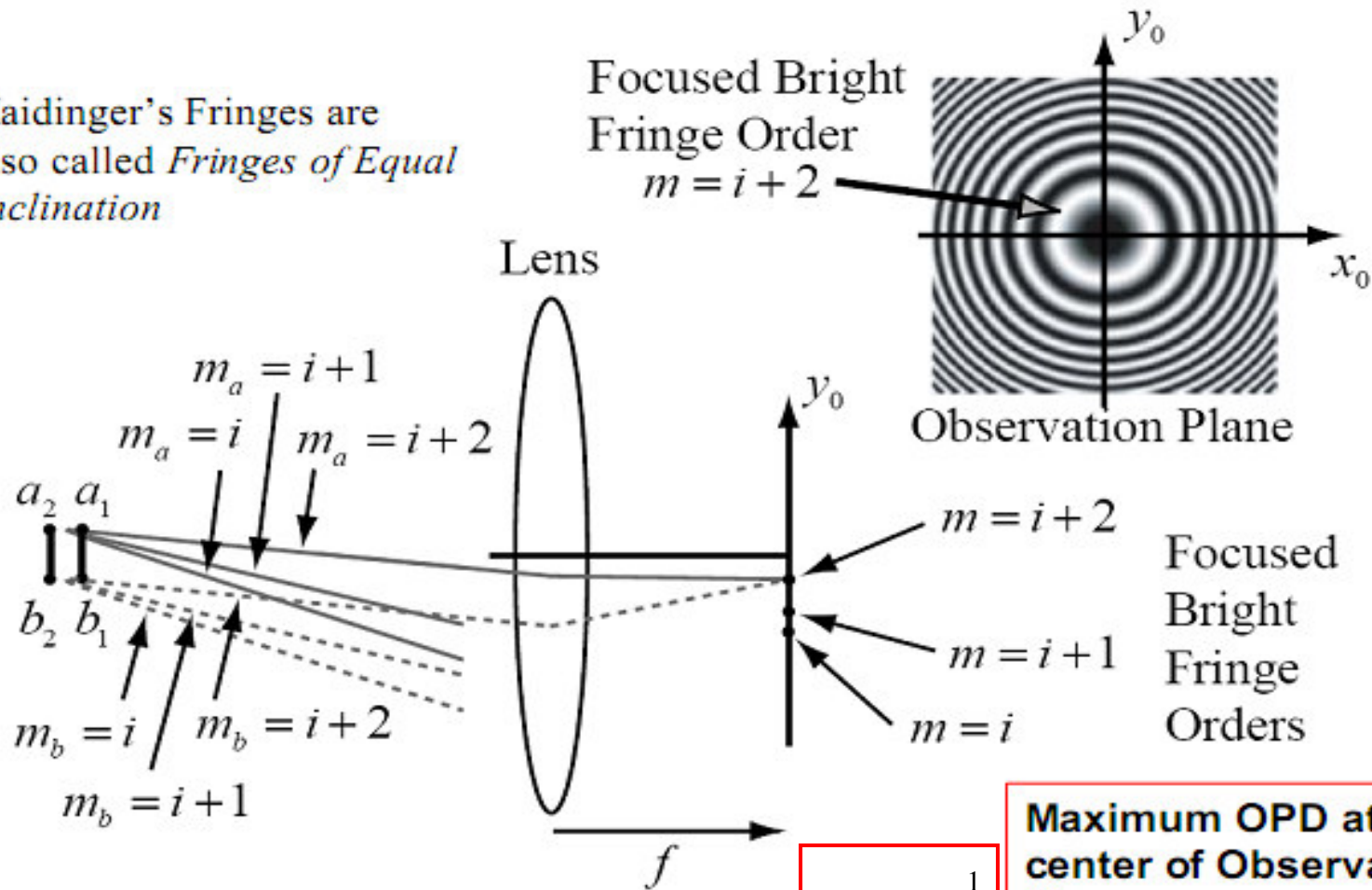
Theory: Fringes of Equal Inclination

- For “thick” thin films, we find fringes of equal inclination by viewing the film at near-normal incidence (viewing screen parallel to film)
 - Called **Haidinger Fringes** after Austrian physicist Wilhelm Karl Haidinger (1795-1871)



Theory: Fringes of Equal Inclination

Haidinger's Fringes are also called *Fringes of Equal Inclination*



Maximum OPD at center of Observation Plane

$$\theta_p = \left(\frac{p \cdot \lambda_0}{d} \right)^{\frac{1}{2}}$$

Objectives

- Record the position of the micrometer after every 100 whole fringes pass through a specific point
- Plot mirror displacement Vs. change in micrometer reading. Slope is equal to a conversion factor to be implemented in the determination of the film thickness

- Determine the film thickness by using the following formulas:
$$(n-1)t = d_1$$
$$nt = d_2$$

n = index of refraction of film
t = film thickness
d = distance from reference point to color fringe pattern on the film

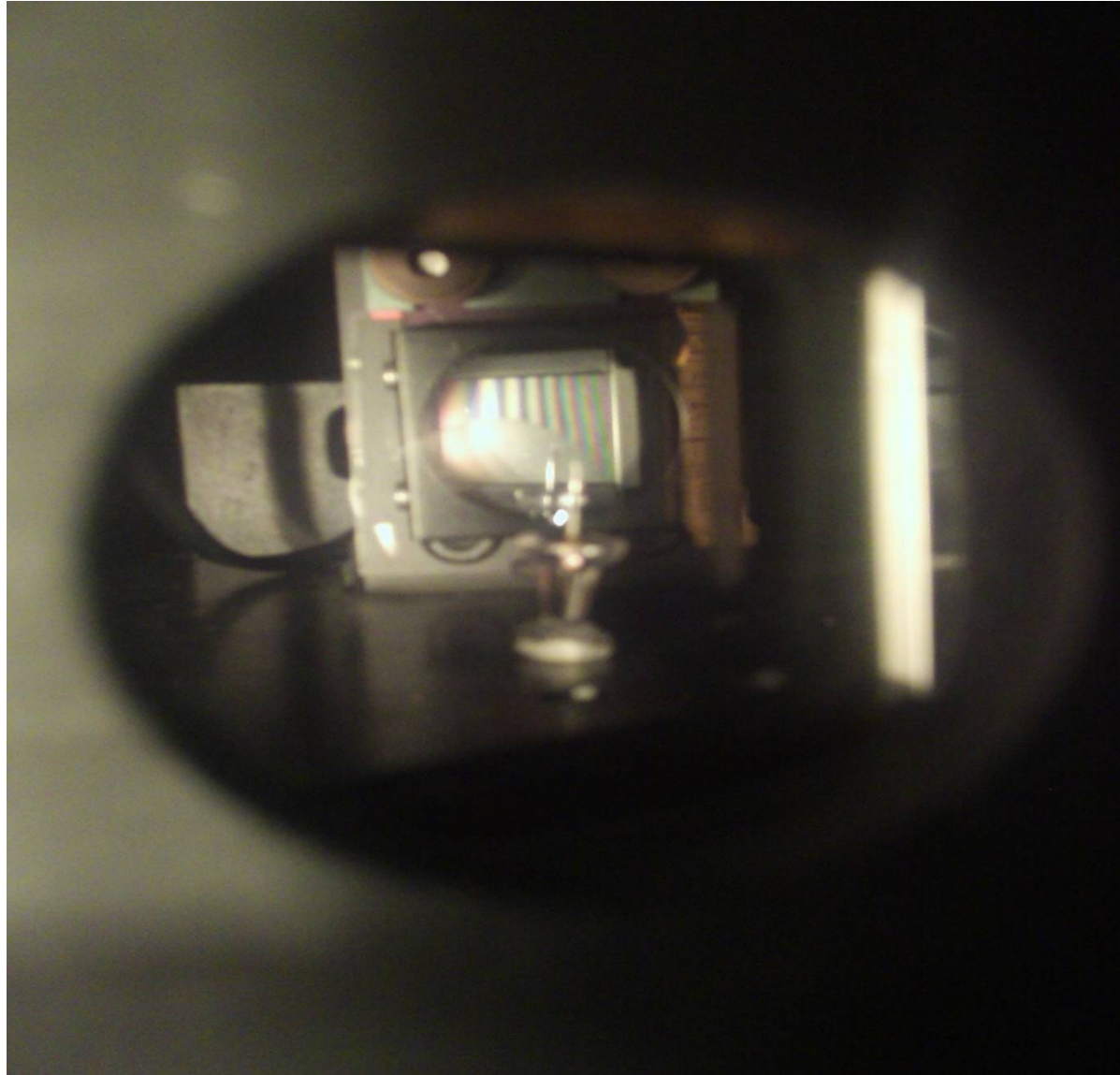
Procedure

- Use Michelson Interferometer to center the fringe pattern using a laser as our source
- Choose a reference point on the pattern and count a total of 1000-2000 fringes that pass by displacing the mirror using the micrometer.
- Plot the mirror displacement vs. the change in the micrometer reading. Slope will be a straight line. It is a calibration factor for the interferometer and converts micrometer displacement to mirror displacement.

Procedure

- Move mirrors until only 2-3 fringes are visible.
- Turn micrometer until there is a change in curvature of the fringe pattern.
- This is the point where the fringes appear as straight lines, and where the white light fringes will appear.
- Locate the colored fringe pattern on the thin film by observing this pattern through the detector.

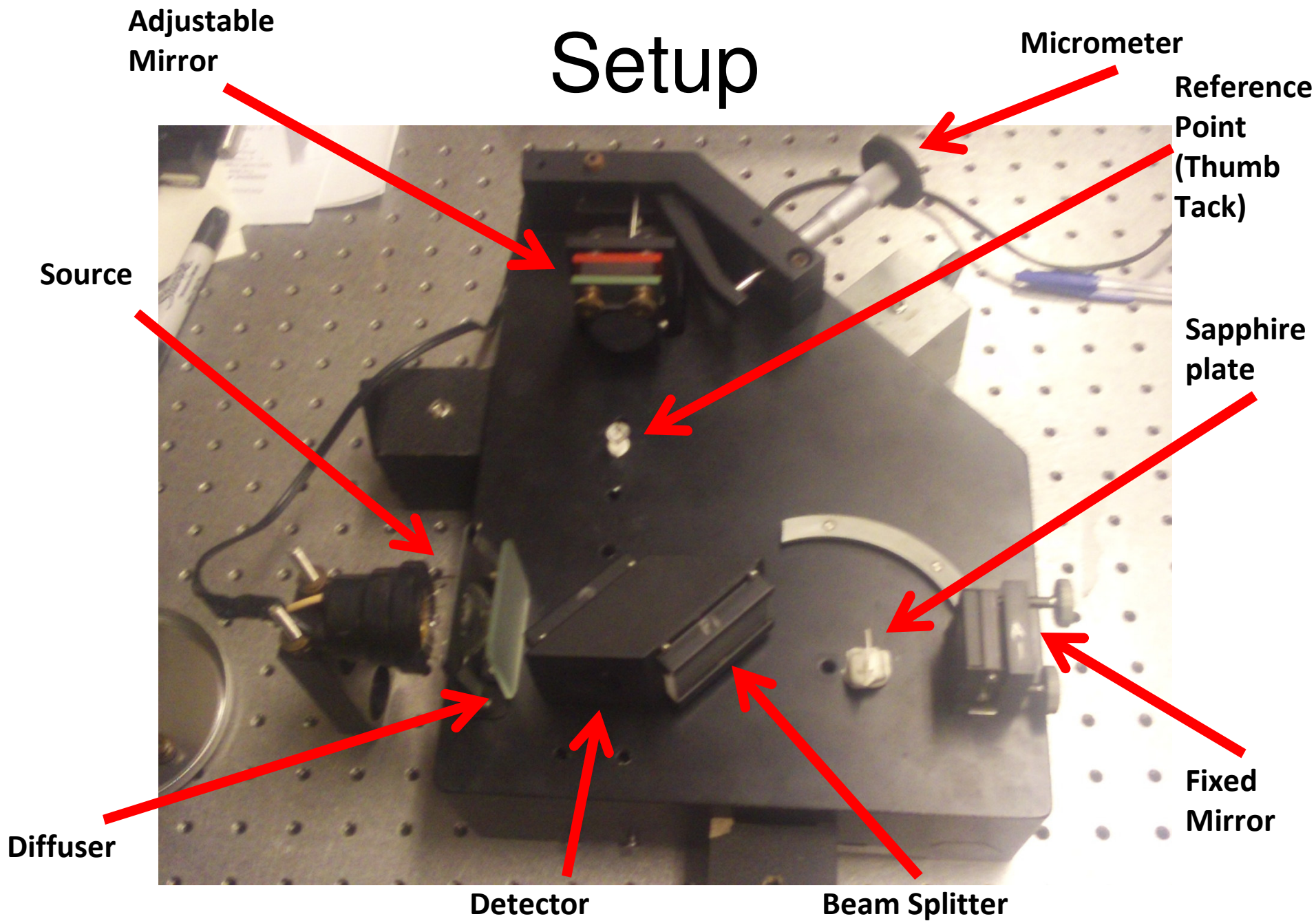
Procedure



Procedure

- Insert the sapphire plate in the adjustable leg with clay.
- Locate the colored fringe pattern on the film caused by the optical path difference (OPD) due to the plate.
- Record the displacement needed for this colored fringe. There are two colored fringes caused by the OPD. One at a distance d_1 and one at d_2 . The fringe pattern at d_2 is extremely difficult to see.

Setup



Setup

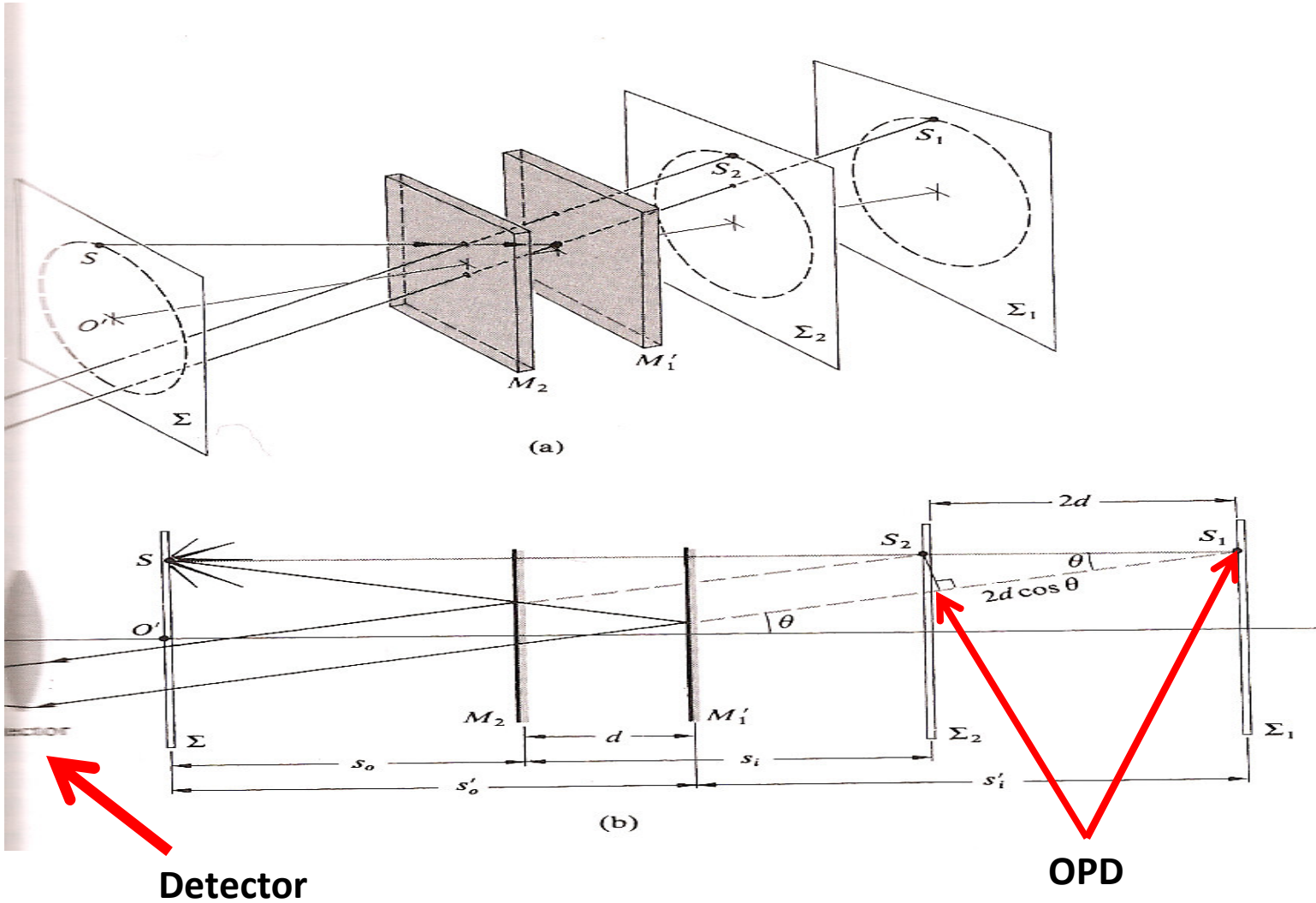
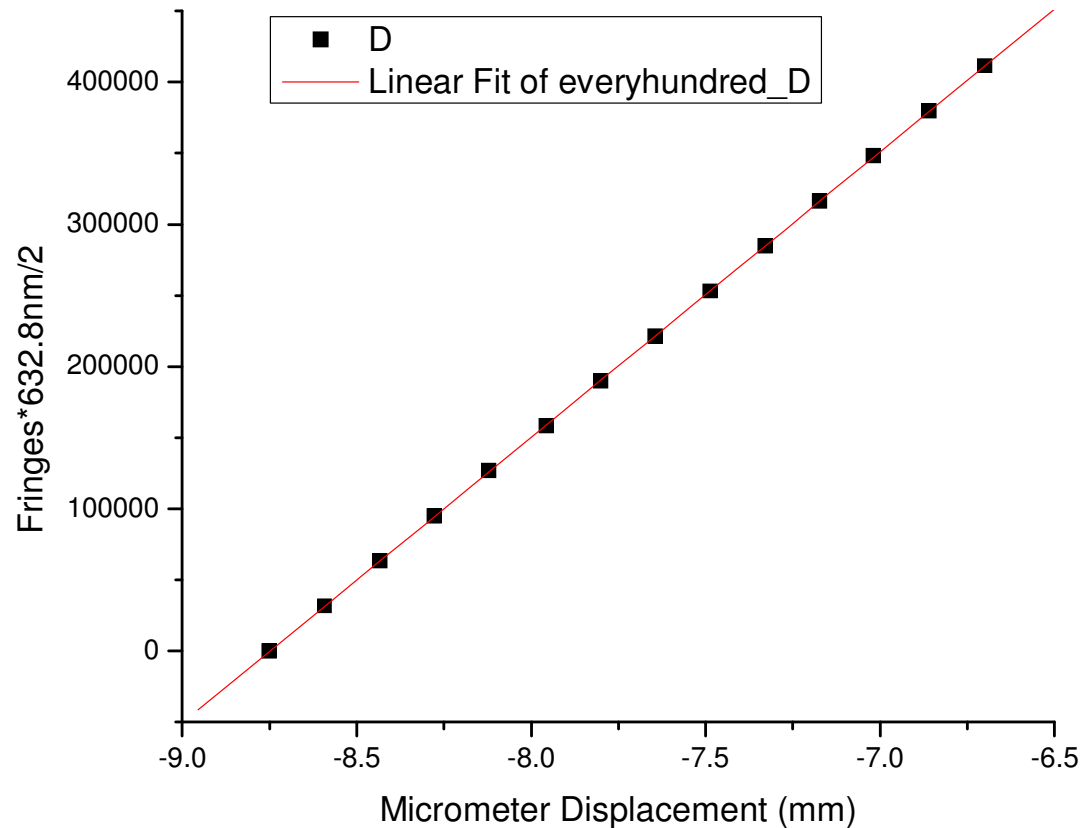


Figure 9.25
rearranger
Interferomete

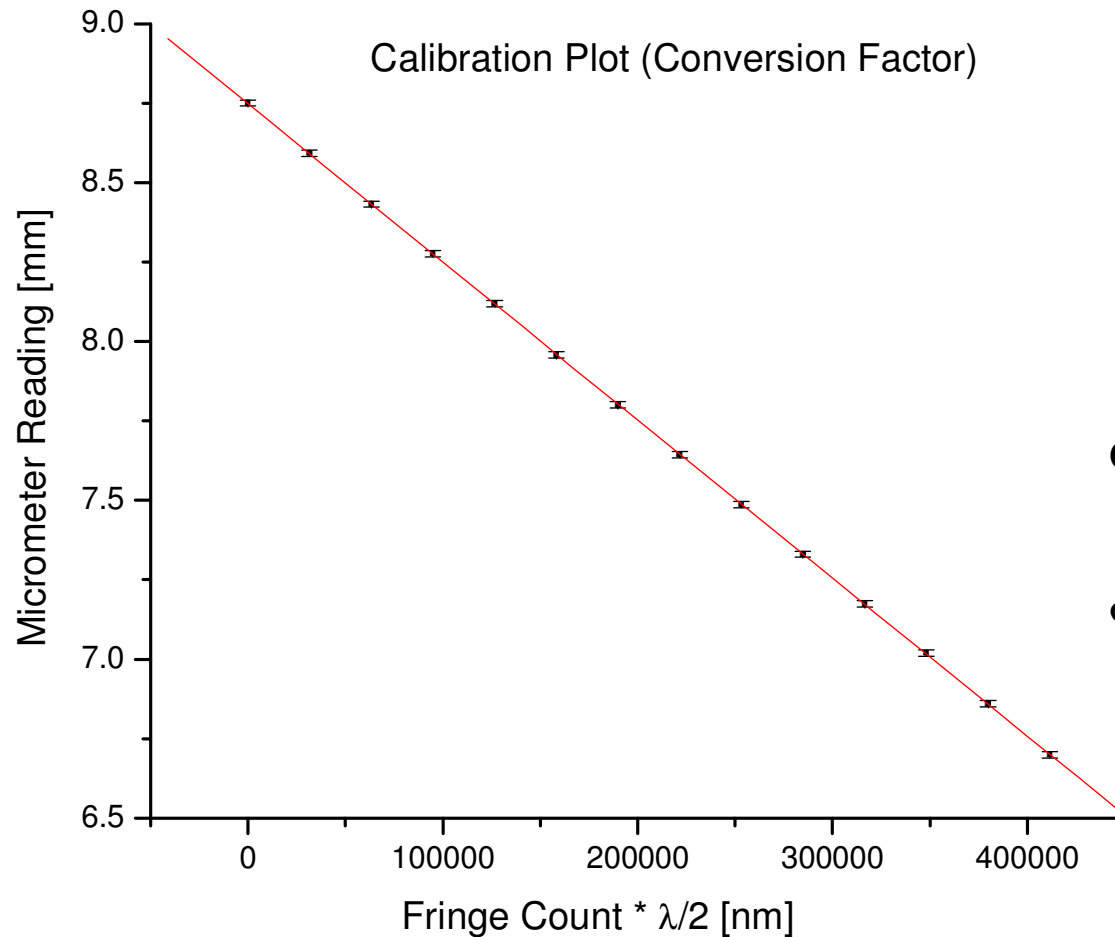
Procedure Alignment

Move 100 fringes and record micrometer displacement. plot fringes*632.8nm/2 vs micrometer reading and a correction factor is obtained to use in the visible light part of this lab.

Slope translates micrometer movement into pathlength difference in Michelson system.



Calibration Plot with Error Estimates



$$Y = A + B \cdot X$$

$$A = (8.74811 \pm 0.00507) \cdot \text{mm}$$

$$B = (-4.97741\text{E-}6 \pm 2.09536\text{E-}8) \cdot \frac{\text{mm}}{\text{nm}}$$

$$\text{ConvFactor} := \left| B^{-1} \right| = 2.009 \times 10^5 \frac{\text{nm}}{\text{mm}}$$

$$\sigma_{\text{ConvFactor}} := \frac{1}{B^2} \cdot \sigma_B = 845.769 \frac{\text{nm}}{\text{mm}}$$

Issues With Experimental Observations

- The way the manual is written, the d1 would be presented before d2
- Apparently this is not the case; due to our not seeing the real d1 phenomenon first and mistaking d2 for d1
- Once these are switched around, the index of refraction is very close to expected value

Results

in air lines

$$\text{line} := \begin{pmatrix} 13.065 \\ 13.083 \\ 13.09 \\ 13.07 \\ 13.07 \\ 13.065 \end{pmatrix} \text{mm}$$

first interference
pattern observed

$$\text{line}_1 := \begin{pmatrix} 11.04 \\ 11.055 \\ 11.035 \\ 11.025 \\ 11.04 \\ 15.10 \end{pmatrix} \text{mm}$$

second interference
pattern observed

$$\text{line}_2 := \begin{pmatrix} 9.035 \\ 8.955 \\ 8.995 \\ 8.985 \\ 9.02 \\ 17.11 \end{pmatrix} \text{mm}$$

The first five are shrinking OPD, micrometer mirror moving in. The sixth is OPD enlarging, micrometer mirror moving out. Important to note, mirror moving out had less observed intensity than going in. In both directions however, constructive interference was observed.

Results

$$t = \frac{d}{n - 1}$$

$$d = \text{Reading} \cdot \text{ConvFactor}$$

$$n := 1.76$$

$$\text{Reading} := \text{line}_2 - \text{line} = \begin{pmatrix} -4.03 \\ -4.128 \\ -4.095 \\ -4.085 \\ -4.05 \\ 4.045 \end{pmatrix} \text{ mm}$$

$$t := \overrightarrow{\left| \frac{d}{n - 1} \right|} = \begin{pmatrix} 1.065 \\ 1.091 \\ 1.083 \\ 1.08 \\ 1.071 \\ 1.069 \end{pmatrix} \text{ mm}$$

$$t_{\text{theo}} = (1 \pm 0.05) \text{ mm}$$

$$d := \text{Reading} \cdot \text{ConvFactor} = \begin{pmatrix} -0.81 \\ -0.829 \\ -0.823 \\ -0.821 \\ -0.814 \\ 0.813 \end{pmatrix} \text{ mm}$$

$$\%_{\text{error}} := \frac{t - t_{\text{theo}}}{t_{\text{theo}}} = \begin{pmatrix} 6.534 \\ 9.125 \\ 8.252 \\ 7.988 \\ 7.063 \\ 6.93 \end{pmatrix} \%$$

ERROR ANALYSIS

$$t = \frac{d}{n - 1} \quad \sigma_t = \frac{1}{n - 1} \cdot \sigma_d$$

$$d = \text{Reading} \cdot \text{ConvFactor} \quad \sigma_d = \sqrt{(\text{Reading} \cdot \sigma_{\text{ConvFactor}})^2 + (\text{ConvFactor} \cdot \sigma_{\text{Reading}})^2}$$

$$\sigma_{\text{Reading}} := 0.01 \text{mm}$$

$$\sigma_d := \sqrt{(\text{Reading} \cdot \sigma_{\text{ConvFactor}})^2 + (\text{ConvFactor} \cdot \sigma_{\text{Reading}})^2} = \begin{pmatrix} 3.957 \times 10^{-3} \\ 4.028 \times 10^{-3} \\ 4.004 \times 10^{-3} \\ 3.997 \times 10^{-3} \\ 3.971 \times 10^{-3} \\ 3.967 \times 10^{-3} \end{pmatrix} \text{mm}$$

$$\sigma_t := \frac{1}{n - 1} \cdot \sigma_d = \begin{pmatrix} 5.206 \times 10^{-3} \\ 5.3 \times 10^{-3} \\ 5.268 \times 10^{-3} \\ 5.259 \times 10^{-3} \\ 5.225 \times 10^{-3} \\ 5.22 \times 10^{-3} \end{pmatrix} \text{mm}$$

$$t = \begin{pmatrix} 1.065 \\ 1.091 \\ 1.083 \\ 1.08 \\ 1.071 \\ 1.069 \end{pmatrix} \text{mm} \quad +/-$$

$$\begin{pmatrix} 5.206 \times 10^{-3} \\ 5.3 \times 10^{-3} \\ 5.268 \times 10^{-3} \\ 5.259 \times 10^{-3} \\ 5.225 \times 10^{-3} \\ 5.22 \times 10^{-3} \end{pmatrix} \text{mm}$$

More ERROR ANALYSIS

$$N := 6 \quad \Delta := \left[N \cdot \sum_{i=0}^{N-1} (x_i)^2 \right] - \left(\sum_{i=0}^{N-1} x_i \right)^2 = 1.051 \times 10^{-7} \text{ m}^2$$

$$A_2 := \frac{\left[- \sum_{i=0}^{N-1} (x_i) \right] \left[\sum_{i=0}^{N-1} (x_i \cdot y_i) \right] + \left[\sum_{i=0}^{N-1} (x_i)^2 \right] \left(\sum_{i=0}^{N-1} y_i \right)}{\Delta} = 8.75 \times 10^{-3} \text{ m}$$

$$B_2 := \frac{N \cdot \sum_{i=0}^{N-1} (x_i \cdot y_i) - \left(\sum_{i=0}^{N-1} x_i \right) \cdot \left(\sum_{i=0}^{N-1} y_i \right)}{\Delta} = -4.999$$

$$A = 8.748 \text{ mm} \quad A_2 = 8.75 \text{ mm}$$

$$B = -4.977 \quad B_2 = -4.999$$

$$\chi^2 := \sum_{i=0}^{N-1} \left(\frac{y_i - A - B \cdot x_i}{\sigma_{\text{Reading}_i}} \right)^2 = 0.254$$

Conclusions

- Our value for the index of refraction is close to expected value when we switch d1 and d2. The index of refraction for the other value is not close. What we need is an equation for d3 to plug in the data collected and compare to d2 index of refraction.

$$n_1 = 1.818 \pm 0.010$$

$$n_2 = .409 \pm 5.475 \cdot 10^{-3}$$

n for sapphire glass is around 1.76 [1]

Percent Error

$$\frac{\text{mean}(n_1) - n_{\text{expected}}}{n_{\text{expected}}} \cdot 100 = 3.302$$

$$\frac{|\text{mean}(n_2) - n_{\text{expected}}|}{n_{\text{expected}}} \cdot 100 = 76.755$$

Observations

- Small sapphire plate, 13mmX.5mm did not reveal any type of interference pattern; could not compare this
- Instrumentation is not delicate enough for extreme precision and adjustment; budget issue
- Any slight movement would misalign system
- No observed “d1” fringe pattern

Sources of Error

- Table, body contributing to vibrations that actually misaligned system
- Reading the micrometer scale: fatigue, eye strain, wrong angle, reading dial directionally wrong
- Not having *perpendicularness* throughout system with mirrors and sapphire plate; additional OPD present
- Convection currents due to white light and air currents in general make interference lines wiggle back and forth. For insane precision this would be very problematic

References

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