Thermal Physics PHY474 Lab #4
Ruchardt Method for Measuring C_p/C_V

We simulate a mechanical experiment used to measure the constant $\gamma = C_p/C_V$. This method was developed by Ruchardt in 1929. The gas is in a container of volume V_0. A tube of cross-sectional area A in which a ball of mass m fits perfectly is part of the container. The ball is in equilibrium when:

$$mg + Ap_{atm} = Ap_0.$$

The falling ball compresses the gas which then expands and pushes the ball up. The oscillations are fast enough for heat not to be exchanged, i.e. the gas undergoes an adiabatic process. We start by writting Newton's second law for the ball:

$$md^2z/dt^2 = -mg - Ap_{atm} + Ap.$$

Assuming the gas to be ideal and the process adiabatic, we can use the Poisson equation: $pV^{\gamma} = p_0V_0^{\gamma}$. Taking z = 0 when the ball is in equilibrium, if the ball is above the equilibrium position then the gas volume is $V = V_0 + Az$. The differential equation for z(t) becomes:

$$d^2z/dt^2 = -g + (A/m)[p_0(1 + Az/V_0)^{-\gamma} - p_{atm}], \quad \text{where } p_0 = p_{atm} + mg/A.$$

If the oscillations amplitude is small we can expand the right hand side of the differential equation in powers of z:

$$d^2z/dt^2 + \omega^2 z = 0 \text{ where the angular velocity is: } \omega = [(A^2\gamma p_0)/(V_0 m)]^{1/2}.$$

So by measuring the period of small oscillations and by using $T = 2\pi/\omega$ one can determine the constant γ:

$$\gamma = 4\pi^2 mV_0/[T^2A^2(p_{atm}+mg/A)].$$ This will be done in the next lab session.

We start the simulation by inputing the information about the ball: m, A and the ga
V_0 , p_{atm} , g.

$$m := 10^{-2} \qquad A := 10^{-4} \qquad \gamma := \frac{7}{5}$$

$$g := 9.8 \qquad V_0 := 10^{-2} \qquad p_{atm} := 10^{5}$$

$$p_0 := p_{atm} + m \cdot \frac{g}{A}$$

The period computed from the small oscillations approximation is:

$$T_{apprx} := 2\cdot\pi\cdot\sqrt{\frac{V_0\cdot m}{A^2\cdot p_0\cdot\gamma}} \qquad T_{apprx} = 1.671$$

We solve the differential equation by using the Mathcad solver Odesolve. You have used this solver in Lab#3. This solver uses either the fixed step 4'th order Runge Kutta method or an adaptive method. Click on Odesolve with the right mouse button and choose Fixed or Adaptive from the pop-up menu. You should get more information on this function by using: Help?, Functions, Built-in functions Differential equations solvers.The solver provises the position of the ball z at N time values in the interval 0 to ttot .

$$N := 1000 \qquad ttot := 10\cdot T_{apprx}$$

Given

$$\frac{d^2}{dt^2}z(t) = -g + \frac{A}{m}\cdot\left[p_0\cdot\left(1 + A\cdot\frac{z(t)}{V_0}\right)^{-\gamma} - p_{atm}\right]$$

The initial values of position and velocity are: $z(0) = 100 \qquad z'(0) = 0$

$$z := \text{Odesolve}(t, ttot, N)$$

Note: to study the effect of the nonlinearity in force it helps to input a very large (though unrealistic) initial z.

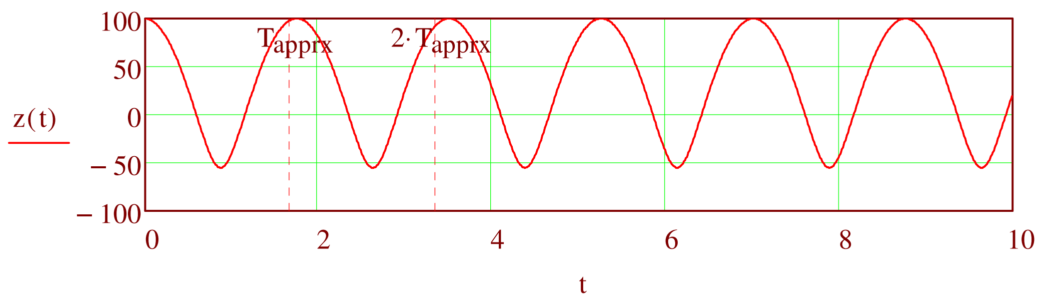
The first derivative of z is velocity: dz/dt = v

$$v(t) := \frac{d}{dt}z(t)$$

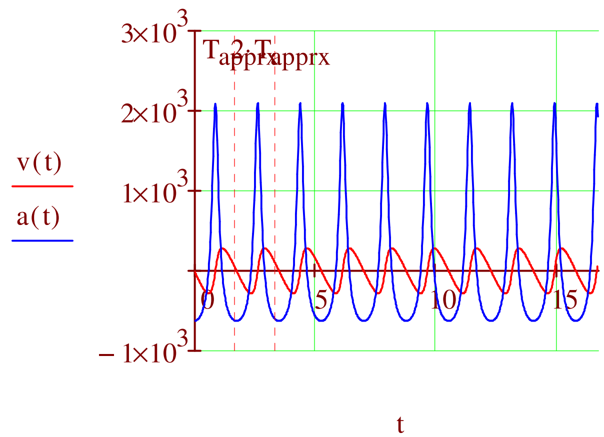
The second derivative is acceleration: $d^2z/dt^2 = F/m$.

$$a(t) := -g + \frac{A}{m}\cdot\left[p_0\cdot\left(1 + A\cdot\frac{z(t)}{V_0}\right)^{-\gamma} - p_{atm}\right]$$

We graph first: position as function of time. We can estimate the period of the oscillations. Note it is longer than T, the period for small oscillations. Also note that the average value of z is positive; the ball spends most of the time above the equilibrium position.



Next we graph velocity and acceleration versus time.



The trajectory in phase space (z,v):