## MTH 155-CALCULUS FOUR (CL4): SAMPLE EXAM

If $P(x,y)$ is a function of $x$ and $y$ then $P_x(x,y)$ is the partial derivative with respect to $x$. $P_{xy}(x,y)$ is the second partial derivative of $P$ with respect to $x$, with respect to $y$, etc.

Recall: Suppose that $f(x,y)$ is a function such that $f_{xx}, f_{yy}, f_{xy}$ all exist. Let $(a,b)$ be a critical point for which:

$$f_x(a,b) = 0 \text{ and } f_y(a,b) = 0$$

Let $M$ be the number defined by

$$M = f_{xx}(a,b) \cdot f_{yy}(a,b) - [f_{xy}(a,b)]^2$$

If $M > 0$ and $F_{xx}(a,b) < 0$, then $f$ has a local maximum at $(a,b)$.
If $M > 0$ and $F_{xx}(a,b) > 0$, then $f$ has a local minimum at $(a,b)$.
If $M < 0$, then $f$ has a saddle-point at $(a,b)$.
If $M = 0$, the test gives no information.

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**PROBLEM 1.** Let $P(x,y) = \ln(x^2y)$. Then $P_x(1,1)$ is closest to

| | | | | |
|---|---|---|---|---|
| **A.** $-3$ | **B.** $-2$ | **C.** $-1$ | **D.** $0$ | **E.** $1$ |
| **F.** $2$ | **G.** $3$ | **H.** $4$ | **I.** $5$ | **J.** $6$ |

**PROBLEM 2.** and $P_{xy}(1,3)$ is closest to

| | | | | |
|---|---|---|---|---|
| **A.** $-4$ | **B.** $-3$ | **C.** $-2$ | **D.** $-1$ | **E.** $0$ |
| **F.** $1$ | **G.** $2$ | **H.** $3$ | **I.** $4$ | **J.** $5$ |

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**PROBLEM 3.** Find the critical points for the function $f(x,y) = 7x^2 - 5xy + 4y^2 + 128x - 83y + 97$. Then the x-coordinate of the critical point is closest to

| | | | | |
|---|---|---|---|---|
| **A.** $-7$ | **B.** $-6$ | **C.** $-5$ | **D.** $-4$ | **E.** $-3$ |
| **F.** $-2$ | **G.** $-1$ | **H.** $0$ | **I.** $1$ | **J.** $2$ |

**PROBLEM 4.** And the y-coordinate of the critical point is closest to

| | | | | |
|---|---|---|---|---|
| **A.** $-1$ | **B.** $0$ | **C.** $1$ | **D.** $2$ | **E.** $3$ |
| **F.** $4$ | **G.** $5$ | **H.** $6$ | **I.** $7$ | **J.** $8$ |

**PROBLEM 5.** Mark the number closest to the extreme value. If the point is a saddle point, then 'saddle point' should be your answer.

| | | | | |
|---|---|---|---|---|
| **A.** $-900$ | **B.** $-800$ | **C.** $-700$ | **D.** $-600$ | **E.** $-500$ |
| **F.** $-400$ | **G.** $-300$ | **H.** $-200$ | **I.** $-100$ | **J.** saddle point |

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**PROBLEM 6.** Find the x-coordinates where horizontal tangents for $f(x) = 2x^3 - 3x^2 - 19$ occur. The rightmost one occurs closest to

| | | | | |
|---|---|---|---|---|
| **A.** $-5$ | **B.** $-4$ | **C.** $-3$ | **D.** $-2$ | **E.** $-1$ |
| **F.** $0$ | **G.** $1$ | **H.** $2$ | **I.** $3$ | **J.** $4$ |

**EXAM CONTINUES ON BACK OF SHEET**

**PROBLEM 7.** The supply and demand functions for a certain commodity are given by: supply: $p = 2 + \frac{9x}{4}$, and demand: $p = 13 - \frac{x}{2}$. Then producers' surplus is closest to

| | | | | |
|---|---|---|---|---|
| **A.** 16 | **B.** 17 | **C.** 18 | **D.** 19 | **E.** 20 |
| **F.** 21 | **G.** 22 | **H.** 23 | **I.** 24 | **J.** 25 |

**PROBLEM 8.** And consumers' surplus is closest to

| | | | | |
|---|---|---|---|---|
| **A.** 2.75 | **B.** 3.00 | **C.** 3.25 | **D.** 3.50 | **E.** 3.75 |
| **F.** 4.00 | **G.** 4.25 | **H.** 4.50 | **I.** 4.75 | **J.** 5.00 |

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**PROBLEM 9.** Suppose the marginal cost of producing $x$ copies of a book is given by $\frac{dy}{dx} = \frac{1000}{y\sqrt{x}}$. If $y = 200$ when $x = 4$, then the cost of producing 625 books is closest to

| | | | | |
|---|---|---|---|---|
| **A.** 360 | **B.** 390 | **C.** 420 | **D.** 450 | **E.** 480 |
| **F.** 510 | **G.** 540 | **H.** 570 | **I.** 600 | **J.** 630 |

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**PROBLEM 10.** The marginal cost function for Janes Coffee Co. is $MC(q) = 25 - q$, where $q$ is the number of tons of coffee produced. Fixed costs are \$100. The marginal revenue function is $MR(q) = 40 - 2q$. The value of the profit function $P(q)$ when $q = 14$ is closest to

| | | | | |
|---|---|---|---|---|
| **A.** 6 | **B.** 8 | **C.** 10 | **D.** 12 | **E.** 14 |
| **F.** 16 | **G.** 18 | **H.** 20 | **I.** 22 | **J.** 24 |

**PROBLEM 11.** The average cost when $q = 12$ is closest to

| | | | | |
|---|---|---|---|---|
| **A.** 6 | **B.** 9 | **C.** 12 | **D.** 15 | **E.** 18 |
| **F.** 21 | **G.** 24 | **H.** 27 | **I.** 30 | **J.** 33 |

**PROBLEM 12.** The change in revenue if sales increase from 5 to 15 tons is closest to

| | | | | |
|---|---|---|---|---|
| **A.** 120 | **B.** 130 | **C.** 140 | **D.** 150 | **E.** 160 |
| **F.** 170 | **G.** 180 | **H.** 190 | **I.** 200 | **J.** 210 |

The correct answers are: 1-F, 2-E, 3-A, 4-H, 5-D, 6-G, 7-C, 8-F, 9-A, 10-D, 11-H, 12-I