

Automorphisms of polynomial and free algebras

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Abstract. Let $K = F[x_1, \dots, x_n]$ be the polynomial algebra in n variables. Given a non-zero constant $a \in F$ and a polynomial $f \in F[x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_n]$ the map $\tau : K \rightarrow K$ which sends x_i to $ax_i + f$ and leaves all the other variables is clearly invertible, and is called an *elementary* automorphism of K .

The *tame* automorphisms are the ones obtained from elementary using composition. It is known that for $n = 2$ all automorphisms of K are tame.

In 1972 Nagata constructed an automorphism of $K = F[x, y, z]$ which he conjectured is *wild* (i.e. non-tame). This conjecture was standing for over 30 years until it was proved by Shestakov and Umirbaev's in 2003. A similar long-standing question regarding the Anick automorphism of the free associative algebra in three variables was recently settled by Umirbaev.

I will give a survey of the results of the last several years related to these questions.