

THE FIRST ANNUAL CSU FRESHMAN-SOPHOMORE
MATHEMATICS COMPETITION

Saturday, April 21, 2007
12:00 noon in RT-1516

Box lunches will be provided by the Mathematics Department.

First Prize: \$250
Second Prize: \$150
Third Prize: \$100

This competition is open to all regularly enrolled CSU students who have taken **at most one** mathematics course numbered above 300.

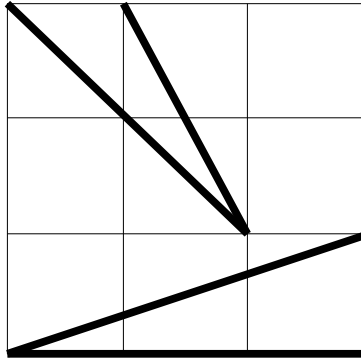
Sign up in the Mathematics Department, RT-1515, by **5:00 p.m. on Monday, April 16.**

The background for the exam is basic algebra and geometry, very elementary number theory, and calculus; no upper-division coursework is necessary. Here are some sample problems to show roughly what you can expect.

1. What is the smallest possible area of a triangle whose sides are formed by the positive x and y axes and a line through the point $(1, 2)$?
2. Find $\int_{-1}^1 \frac{e^{1/x} dx}{x^2(1+e^{1/x})}$.
3. Prove that for any positive integer n , the expression $1110^n + 1102^n - 200^n - 10^n$ is divisible by 2002.
4. The $(5, 12, 13)$ right triangle has area and perimeter both equal to 30. Find all right triangles T with integral sides such that the area of T equals the perimeter of T .
5. A function f is *odd* if $f(-x) = -f(x)$ for all x . Prove that $f(x) = \ln(x + \sqrt{x^2 + 1})$ is odd.
6. Show that there are infinitely many squares that are the sum of a square and a prime.
7. Find a polynomial P with $P(1) = 1$, $P(2) = 2$, and $P(3) = 2003$.
8. If $f(x) = x^2 e^x$, find a formula for the n -th derivative of f .
9. Let $R(t)$ be the area of the region bounded by the y -axis, a positive continuous function $f(x)$, a negative continuous function $g(x)$, and the line $x = t^4$. Compute $R'(2)$.

(Continued on back.)

10. An 'hourglass' is formed by rotating the graph of $y=e^x$ about the line $y=x$. Find the largest tangent sphere that can be passed through the neck of the hourglass.
11. (a) Show that $x/y+y/z+z/x=1$ has no solution in positive integers.
 (b) Find a solution to $x/y+y/z+z/x=5$ in positive integers.
12. Let $\lfloor x \rfloor$ be the greatest integer less than or equal to x . For example, $\lfloor \pi \rfloor=3$, and $\lfloor -\pi \rfloor=-4$. Sketch the graphs of the following:
- (a) $y=\lfloor x \rfloor$
 (b) $\lfloor y \rfloor=\lfloor x \rfloor$
 (c) $\lfloor y \rfloor=\lfloor \lfloor x \rfloor \rfloor$
13. Show that the acute angles formed by the boldface V's in the figure below are equal. (The sketch is imperfect; assume that the grid really is a square consisting of nine small squares.)



Contest Committee: Ivan Soprunov, Brian M. Scott