Non-Equilibrium Dynamics in the Evolution of Industries: A Computational Study*

Myong-Hun Chang
Department of Economics
Cleveland State University
Cleveland, OH 44115
216-687-4523, -9206 (fax)
m.chang@csuohio.edu
http://academic.csuohio.edu/changm/

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Abstract

This study attempts to explain many well-documented aspects of the evolution of industries over time. It uses an agent-based computational model in which artificial industries are created and grown to maturity in silico. While the firms in the model are assumed to have bounded rationality, they are nevertheless adaptive in the sense that their experience-based R&D efforts allow them to search for improved technologies. Given a technological environment subject to persistent and unexpected external shocks, the computationally-generated industry remains in a perennial state of flux. The main objective of the study is to identify patterns that exist in the movements of firms as the industry evolves over time along the steady state in which the measured behavior of the firms and the industry stochastically fluctuate around steady means. The computational model developed here is able to replicate many of the stylized facts from the empirical industrial organization literature, particularly as the facts pertain to the dynamics of firm entry and exit. Furthermore, the model allows examination of cross-industry variations in entry and exit patterns by systematically varying the characteristics of the market and the technological environment within which the computationally-generated industry evolves. The model demonstrates that the computational approach based on boundedly rational agents in a dynamic setting can be useful and effective in carrying out both positive and normative economic analyses.

* This is a condensed version of the manuscript for a forthcoming book, A Computational Model of Industry Dynamics (Routledge). Please do not circulate without the author’s permission.
The evolution of market structure is a complex phenomenon and the quest for any single model that encompasses all the statistical regularities observed is probably not an appropriate goal. Yet there remain phenomena which may well be worth encompassing in a more general theory than is currently available, and which are still poorly understood. Most notable among these are questions of the industry-specific determinants of firm turnover (turbulence) and the volatility of market shares. Another such area is that of the pattern of exit in declining industries. Notwithstanding recent progress on these topics, many important questions still remain open.


1. INTRODUCTION

This research was motivated by four stylized facts in empirical Industrial Organization (IO). The first is the phenomenon of shakeout in infant industries. This widely reported phenomenon describes the sudden inflow of firms at the birth of an industry, followed by a rapid outflow of firms that bears a striking resemblance to a market crash. This is a phenomenon that has been documented extensively.¹ The second is the observation that the entries and exits of firms persist in the long run even in mature industries. Third, the movements into and out of the industry tend to occur together when they occur: a period of high rate of entry is also the one of a high rate of exit. Finally, the severity of such structural turbulence tends to differ across industries: Some industries are characterized by high rates of entry and exit, while others are characterized by relatively low rates of entry and exit. This calls for a systematic study of how the shape, size, and nature of the industry dynamics are determined by the industry-specific factors:

“A last observation concerns the enormous variation across new industries in the pace and severity of the prototypical pattern of industry evolution. This suggests that there are important differences across industries in the factors that condition the evolutionary process.” [Klepper and Graddy (1990)]

“[W]e find substantial and persistent differences in entry and exit rates across industries. Entry and exit rates at a point in time are also highly correlated across industries, so that industries with higher than average entry rates tend to also have higher than average exit rates. Together these suggest that industry-specific factors play an important role in determining entry and exit patterns.” [Dunne, Roberts, and Samuelson (1988)]

In this paper, I present a general model of industry dynamics that allows one to perform a wide variety of computational experiments, successfully replicate the stylized facts mentioned above, and make additional predictions based on the underlying theory of firm behavior and market competition. The model is motivated by a couple of initial impressions on what may constitute the causal mechanism driving the dynamics of firms and industries. First, the dramatic rise and fall in the number of firms during a shakeout suggests a degree of myopia on the part of entrants. Second, business firms grapple with unexpected shocks to their operating environment on a daily basis – shocks that create new profit opportunities for some but simultaneously drive others to extinction.

Such non-uniform effects of unanticipated external shocks can induce correlations between the entry and exit rates of firms across time (i.e., a period with an above-average rate of entry also has an above-average rate of exit) and across industries (i.e., the industry with an above-average rate of entry also has an above-average rate of exit).

IO theorists have used various analytical and numerical approaches to explore the entry and exit dynamics of firms as well as their impacts on the growth of the industry. [Jovanovic (1982), Hopenhayn (1992), Asplund and Nocke (2006)] They endow the firms, as is standard in current economic theory, with perfect rationality and foresight such that the entry and exit decisions are made to maximize the expected discounted value of future net cash flow. The solution concept typically entails stationary equilibrium which characterizes the behavior of firms in the steady state in the presence of productivity shocks. Given the dynamic nature of the framework involving heterogeneous firms, the scale and scope of investigations tend to be limited in these models due to either the restrictive nature of the assumptions used to foster tractability, or the formidable analytical difficulties resulting from less restrictive assumptions.

The computational model developed here eschews the standard behavioral assumptions in the tradition of “orthodox” economic theory. Instead, the dynamic patterns arising at the level of an industry are viewed as the result of continuing interactions among myopic and heterogeneous firms who are motivated by profits and engaged in R&D as a way to improve their profits. The use of bounded rationality is deemed appropriate in this research for two reasons. The first reason is that the central issue under investigation deals with market conditions that are highly stochastic and thus lead to inherently turbulent market structure. In the context of our model, these market conditions include the following: 1) The technological environment is subject to continual external shocks which affect the efficiencies of the operating firms randomly across time and asymmetrically across firms; and 2) firms with heterogeneous technologies enter and exit the market each period, thus inducing persistent volatility in the structure of the market. These factors lead to a decision environment that is fraught with uncertainty and incomplete information. In this environment a rational pursuit of expected profit maximization is made difficult by the limited ability of a decision maker to predict the inter-temporal movements of the variables that determine the future states, and it is appropriate to make the assumption of bounded rationality.

The second reason for limiting the degree of rationality is more practical. As modeled here, firms with bounded rationality use fixed decision rules. Fixed decisions rules substantially reduce the demand for computational resources required in the dynamic optimization process. In the present context, one in which a large number of firms with heterogeneous technologies engage in market interactions while facing a technological environment subject to persistent random shocks, the consequence of assuming perfect rationality is that each firm must solve a stochastic dynamic programming problem in which all possible future states of its rivals conditional on those shocks are fully incorporated (given that the firm holds well-defined subjective beliefs over these shocks and these beliefs are common knowledge to all firms in the market). This process presents a

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2 See Dawid (2006) for a survey of computational papers that take a similar perspective. Tesfatsion and Judd (2006) offers a comprehensive collection of similar surveys that cover the agent-based computational economics (ACE) models in a wide variety of areas in economics.

3 See Dosi and Egidi (1991) for related discussions on “subjective uncertainty” and “procedural uncertainty.”

4 The Markov-perfect equilibrium (MPE) models take a numerical approach to studying the equilibrium firm behavior, while remaining strictly within the dynamic optimization framework. [Pakes and McGuire (1994), Ericson and Pakes (1995)] Although the MPE approach is “numerical” and, hence, designed to avoid the issue of analytical tractability, it suffers from the “curse of dimensionality” which comes with the exponential growth in the size of the state space resulting from the increase in the number of variables. See
computational problem of significant complexity, and it is difficult, if not impossible, for the modeler to keep the model sufficiently rich in detail to generate predictions that can fit the observed data while retaining its analytical tractability. The use of fixed decision rules, by passing over the dynamic optimization process, allows computational resources to be reallocated to tracking and analyzing the adaptive behavior of a realistically large number of firms as they interact with one another over time as the industry grows to its maturity.

The analyses of the time series data, reported in this paper, shed light on the aforementioned stylized facts in the context of the market interactions among competing firms along the transient path of a growing industry. Furthermore, by allowing sufficient time for the industry to grow and mature, the computational platform offers an in-depth look at the adaptive behavior of the firms and the industries along the stochastic steady state, i.e., the eventual state in which the endogenous variable capturing the behavior of firms has a time-independent distribution such that the industry fluctuates stochastically around a steady mean. For ease of exposition, such a state will be referred to simply as “steady state.” To the extent that the industry-specific factors captured by the relevant parameters in the model determine the developmental trajectory of an industry, the computational platform allows cross-industry comparisons of steady states among industries grown under different parameter specifications. This comparative dynamics exercise provides a test of reliability for the model as the computationally generated outcomes can be compared to the widely accepted results from the cross-sectional studies in the empirical IO literature.

The organization of the paper follows. In Section 2, I provide a detailed description of the model. The model is then put to the test using the baseline configuration of the parameters in Section 3. The proto-history for the baseline industry is computationally generated and described in some detail. In Section 4, I take on the topic of shakeout in two parts. First, I consider the typical shakeout in an infant industry by abstracting away from the external shocks to the technological environment. The entry and exit decisions are driven purely by the emergence of the profit opportunity in the newly discovered industry; the entry and exit decisions are not influenced by changes in the technological environment. In the second part of the section I introduce the external shocks and explore the recurrent shakeouts as they are induced by the changes in the technological environments surrounding firms.

The proposed model allows firms to attain steady states in the long run by allowing sufficient time for the industry to grow and mature. In Section 5, I first examine the characteristics of the firm and the industry after the given industry has attained a steady state. I then explore the comparative dynamics by growing industries to maturity under various parameter configurations. Both the intra-industry movements, i.e., changes in the market shares of the firms within the industry, and the inter-industry movements, i.e., entry into and exit out of the industry, of firms are shown to depend on industry-specific factors. The comparative dynamics analysis characterizes the cross-industry differences in volatility in terms of intra- and inter-industry movements.

Doraszelski and Pakes (2007): “The computational burden of computing equilibria is large enough to often limit the type of applied problems that can be analyzed. There are two aspects of the computation that can limit the complexity of the models we analyze; the computer memory required to store the value and policies, and the CPU time required to compute the equilibrium …. If we compute transition probabilities as we usually do using unordered states then the number of states that we need to sum over to compute continuation values grows exponentially in both the number of firms and the number of firm-specific state variables.” [pp. 1915-1916] It should be mentioned that there have been attempts to circumvent this problem while remaining within the general conceptual framework of the MPE approach. See Weintraub, Benkard, and Van Roy (2008, 2010) for recent works in this line of research.
In Section 6, I take a deeper look at the steady state dynamics by observing the endogenous behavior of individual firms along the steady state of a given industry. The resulting variations across firms in terms of their technologies, market shares, and life spans are examined.

Section 7 contains the overall summary and conclusion.

2. THE MODEL

The model entails an evolving population of firms which interact with one another through repeated market competition. Central to this process are the heterogeneous production technologies held by the firms and the R&D mechanism through which they evolve over time.

2.1. Basic Features

2.1.1. Technology

In each period, firms engage in market competition by producing and selling a homogeneous good. The good is produced through a process that consists of $N$ distinct tasks. Each task can be completed using one of two different methods. Even though all firms produce a homogeneous good, they may do so using different combinations of methods for the $N$ component tasks. The method chosen by the firm for a given task is represented by a bit (0 or 1) such that there are two possible methods available for each task and thus $2^N$ variants of the production technology. In period $t$, a firm’s technology is then fully characterized by a binary vector of $N$ dimensions which captures the complete set of methods it uses to produce the good.

Let $z^t_i \in \{0,1\}^N$ denote firm $i$’s technology in period $t$, where $z^t_i \equiv (z^t_i(1), z^t_i(2), ..., z^t_i(N))$ and $z^t_i(h) \in \{0,1\}$ is firm $i$’s chosen method in task $h$. In measuring the degree of heterogeneity between two technologies (i.e., method vectors), $z^t_i$ and $z^t_j$, we use “Hamming Distance,” which is the number of positions for which the corresponding bits differ:

$$D(z^t_i, z^t_j) \equiv \sum_{h=1}^{N} |z^t_i(h) - z^t_j(h)|$$

(1)

The crucial perspective taken in this model is that the efficiency of a given technology depends on the environment it operates in. In order to represent the technological environment that prevails in period $t$, I specify a unique methods vector, $\tilde{z}^t \in \{0,1\}^N$, which is defined as the optimal technology for the industry in $t$. How well a firm’s chosen technology performs in the current environment depends on how close it is to the prevailing optimal technology in the technology space. More specifically, the marginal cost of firm $i$ realized in period $t$ is specified to be a direct function of $D(z^t_i, \tilde{z}^t)$, the Hamming distance between the firm’s chosen technology, $z^t_i$, and the optimal technology, $\tilde{z}^t$. The firms are uninformed about $\tilde{z}^t$ ex ante, but engage in search to get as close to it as possible by observing their marginal costs. The optimal technology is common for all firms – i.e., all firms in a given industry face the same technological environment. As such, once optimal technology is defined for a given industry, its technological environment is completely specified for all firms since the efficiency of any technology is well-defined as a function of its distance to this optimal technology.
I allow turbulence in the technological environment. Such turbulence is assumed to be caused by factors external to the industry in question such as technological innovations that originate from outside the given industry. In a framework closer to the neoclassical production theory, one could view an externally generated innovation as a shock that affects the relative input prices for the firms. If firms, at any given point in time, are using heterogeneous production processes with varying mix of inputs, such a change in input prices will have diverse impact on the relative efficiencies of firms’ production processes – some may benefit from the shock; some may not. Such an external shock will then require (with varying degrees of urgency) a series of adaptive moves by the affected firms for their survival.

The external technology shocks, applied at the beginning of each period, redefine firms’ production environment and such environmental shifts affect the cost positions of the firms in the competitive marketplace by changing the effectiveness of the methods they use in various activities within the production process. These unexpected disruptions then pose renewed challenges for the firms in their efforts to adapt and survive. It is precisely this kind of external shocks that I try to capture in this model. My approach is to allow the optimal technology, \( \hat{\tau}_t \), to vary from one period to the next, where the frequency and the magnitude of its movement represent the degree of turbulence in the technological environment. The exact mechanism through which this is implemented is described in Section 2.2.1.

Finally, in any given period \( t \), the optimal technology is unique. While the possibility of multiple optimal technologies is a potentially interesting issue, it is not explored here because in a turbulent environment, where the optimal technology is constantly changing, it is likely to be of negligible importance. Chang (2009) offered an alternative approach by modeling the technological environment as being stable but with multiple locally optimal technologies. The main focus was on the industry dynamics during the initial shakeout phase, where one of the objectives was to investigate the impact of multiple optima on the shakeout dynamics. In the current study, I am more interested in the dynamics of R&D and firm turnover in the presence of technological turbulence. As such, I abstract away from the possibility of multiple local optima.

### 2.1.2. Demand, Cost, and Competition

In each period, there exists a finite number of firms that operate in the market. In this section, I define the static market equilibrium among such firms. The static market equilibrium defined here is then used to represent the outcome of market competition in each period.

Let \( m^t \) be the number of firms in the market in period \( t \). The firms are Cournot oligopolists, who choose production quantities of a homogeneous good. In defining the Cournot equilibrium in this setting, I assume tentatively that all \( m^t \) firms produce positive quantities in equilibrium. This assumption is made strictly for expositional convenience in this section. In actuality, there is no reason to suppose that in the presence of asymmetric costs all \( m^t \) firms will produce positive quantities in equilibrium. Some of these firms may choose to be inactive by producing zero quantity. The algorithm used to distinguish among active and inactive firms based on their production costs will be addressed in Section 2.2.2.

**Demand**

The inverse market demand function is:

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5 Multiple optima arise naturally when the production process is assumed to be complex – i.e., some or all of the activities are mutually interdependent. [Kauffman(1993)]
\[ P^t(Q^t) = a - \frac{Q^t}{s} \]  \hspace{1cm} \text{(2)}

where \( Q^t = \sum_{j=1}^{m^t} q^t_j \) and \( s \) denotes the size of the market in \( t \). Note that this function can be inverted to \( Q^t = s(a - P^t) \). Hence, for a given market price doubling the market size then doubles the quantity demanded. The demand intercept, \( a \), and the size parameter, \( s \), are assumed to remain fixed over time.

**Cost**

Each firm \( i \) at time \( t \) has its production technology, \( z^t_i \), and faces the following total cost:

\[ C^t(q^t_i) = f + c^t_i \cdot q^t_i \]  \hspace{1cm} \text{(3)}

All firms have identical fixed cost, \( f \), that stays constant over time.

The firm’s marginal cost, \( c^t_i \), depends on how different its technology, \( z^t_i \), is from the optimal technology, \( \hat{z}^t \). Specifically, \( c^t_i \) is defined as follows:

\[ c^t_i(z^t_i, \hat{z}^t) = 100 \cdot \frac{D(z^t_i, \hat{z}^t)}{N}. \]  \hspace{1cm} \text{(4)}

Hence, \( c^t_i \) increases in the Hamming distance between the firm’s chosen technology and the optimal technology for the industry. It is at its minimum of zero when \( z^t_i = \hat{z}^t \) and at its maximum of 100 when all \( N \) bits in the two technologies are different from one another. The total cost is then:

\[ C^t(q^t_i) = f + 100 \cdot \frac{D(z^t_i, \hat{z}^t)}{N} \cdot q^t_i. \]  \hspace{1cm} \text{(5)}

I assume that the technology does not affect the fixed cost.

**Short-Run Market Equilibrium**

Given the demand and cost functions, firm \( i \)’s profit is:

\[ \pi^t_i(q^t_i, Q^t - q^t_i) = \left( a - \frac{1}{s} \sum_{j=1}^{m^t} q^t_j \right) \cdot q^t_i - f - c^t_i \cdot q^t_i. \]  \hspace{1cm} \text{(6)}

Taking the first-order condition for each \( i \) and summing over \( m^t \) firms, we derive the equilibrium industry output rate, which gives us the equilibrium market price, \( \bar{P}^t \), through equation (2):

\[ \bar{P}^t = \left( \frac{1}{m^t + 1} \right) \left( a + \sum_{j=1}^{m^t} c^t_j \right). \]  \hspace{1cm} \text{(7)}

Given the vector of marginal costs defined by the firms’ chosen technologies and the optimal technology, \( \bar{P}^t \) is uniquely determined and is independent of the market size, \( s \). Furthermore, the
equilibrium market price depends only on the sum of the marginal costs and not on the *distribution* of \( c_i \)'s [Bergstrom and Varian (1985)].

The equilibrium firm output rate is:

\[
\tilde{q}_i^t = s \left[ \left( \frac{1}{m^t + 1} \right) \left( a + \sum_{j=1}^{m^t} c_j^t \right) - c_i^t \right].
\]

(8)

Note that \( \tilde{q}_i^t = s \left[ \tilde{p}^t - c_i^t \right] \): A firm’s equilibrium output rate depends on its own marginal cost and the market price. Finally, the Cournot equilibrium firm profit is

\[
\pi^t(\tilde{q}_i^t) = \tilde{p}^t \cdot \tilde{q}_i^t - f - c_i^t \cdot \tilde{q}_i^t = \frac{1}{s} \left( \tilde{q}_i^t \right)^2 - f
\]

(9)

Note that \( \tilde{q}_i^t \) is a function of \( c_i^t \) and \( \sum_{j=1}^{m^t} c_j^t \), where \( c_k^t \) is a function of \( z_k^t \) and \( z_k^t \) for all \( k \). It is then straightforward that the equilibrium firm profit is fully determined, once the vectors of methods are known for all firms. Further note that \( c_i^t \leq c_k^t \) implies \( \tilde{q}_i^t \geq \tilde{q}_k^t \) and, hence, \( \pi^t(\tilde{q}_i^t) \geq \pi^t(\tilde{q}_k^t) \forall i, k \in \{1, ..., m^t\} \).

The use of Cournot-Nash equilibrium is arguably inconsistent with the “bounded rationality” assumption employed throughout the paper. However, explicitly modeling the process of market experimentation would further complicate an already complex model. Therefore, I implicitly assume that experimentation is done instantly and without cost. A Cournot-Nash equilibrium is assumed to be a reasonable approximation of the outcome from that process.\(^6\)

\[ \text{A Diversion: Static Free-Entry Equilibrium} \]

Before discussing the dynamic structure of the model, it is useful to consider what the purely static version of the above Cournot oligopoly model would imply in terms of the long-run industry equilibrium under free entry and exit as it is typically described in textbooks. For this discussion of the static version, we will dispense with the time superscript for all variables. Also assume that the firms are technologically homogenous so that \( c_i^t = c \) for all \( i \in \{1, ..., m\} \). Given \( m \) firms, the symmetric Cournot-Nash equilibrium then entails each firm producing the quantity, \( \overline{q}(m) = \frac{s(a-c)}{m+1} \). The resulting market price is \( \overline{p}(m) = \frac{a+mc}{m+1} \). Each firm earns the equilibrium profit, \( \overline{\pi}(m) = \frac{s(a-c)^2}{(m+1)^2} - f \).

\(^6\) There are results from the experimental economics literature that do support this rather heroic assumption. In their pioneering work, Fouraker and Siegel (1963) conducted experiments with participants who took the role of quantity-adjusting Cournot oligopolists under incomplete information. They found that the Cournot-Nash equilibrium was supported in many trials for the cases of duopoly and triopoly. Similarly, Cox and Walker (1998), using linear demand and constant marginal cost in Cournot duopoly, found that, if a stable equilibrium exists, then the participants in their experiments learn to play the Cournot-Nash equilibrium after only a few periods. Even though the best reply dynamics do not necessarily converge in oligopolies with more than three firms [Theocaris (1960)], Huck et al. (1999) finds that the best reply process does converge if firms are assumed to exhibit some *inertia* in their choice of strategy. For a more general discussion and survey of the literature involving experimental treatment of oligopoly behavior, please see Armstrong and Huck (2010).
Note that the equilibrium profit, \( \bar{\pi}(m) \), decreases in \( m \). The free-entry equilibrium number of firms, \( \bar{m} \), must then satisfy: \( \bar{\pi}(m + 1) < 0 \leq \bar{\pi}(\bar{m}) \). That is, \( \bar{m} \) firms can profitably operate in the industry while \((\bar{m} + 1)\) firms cannot. We may define \( \bar{m} \) as (the integer part of) \( m \) that satisfies \( \bar{\pi}(m) = 0 \), which is:

\[
\bar{m} = (a - c) \sqrt{\frac{c}{f}} - 1.
\]  

(10)

Hence, \( \bar{m} \) is directly related to the market size \( (s) \) and inversely related to the fixed cost \( (f) \): In equilibrium, a larger market has a greater number of firms and a market with higher fixed costs has a smaller number of firms.

How does an industry reach the equilibrium structure defined above? The standard description of the out-of-equilibrium adjustment process involves the following story: For all \( m < \bar{m} \) the incumbent firms earn positive economic profits and this invites entry into the market thus raising \( m \) over time, while for all \( m > \bar{m} \) the incumbents incur economic losses and this induces exits from the market thus reducing \( m \) over time until the losses are eliminated. In the long run, the industry converges to the stable equilibrium structure containing exactly \( \bar{m} \) firms, at which point there is no further entry or exit.

Notice how this implicit mechanism, while guaranteeing the stability of the equilibrium, is incapable of explaining the stylized facts I mentioned earlier: There are no shakeouts on the way to reaching the stable equilibrium; there are no entries and exits once the industry is in equilibrium; entries and exits, even when they occur out of equilibrium, never occur simultaneously; and there is no plausible story as to why the rates of entry and exit should be higher or lower in a given industry when it is out of equilibrium and what are the causal factors.

It is the lack of explanatory capacity in the standard static model of industry that motivates the dynamic model developed in this research. I introduce persistent external shocks to the technological environment of the firms so as to put the out-of-equilibrium process of entry and exit at the center of the analysis. The pursuit of R&D by firms is made endogenous so that technological heterogeneity (and the consequent cost heterogeneity) becomes an essential part of the industry dynamic. The next section describes how these features are implemented in the model.

2.2. Dynamic Structure

In the beginning of any typical period \( t \), the industry opens with two groups of decision makers who face a common market environment as specified by the demand size, \( s \): 1) a group of incumbent firms surviving from \( t - 1 \), each of whom enters \( t \) with a technology, \( z_{t-1} \), and its net wealth, \( w_{t-1} \), carried over from \( t - 1 \); and 2) a group of potential entrants ready to consider entering the industry in \( t \), each with an endowed technology of \( z_t \) and its start-up wealth. All firms face a common technological environment within which his/her technology will be used. This environment is fully represented by the prevailing optimal technology, \( \hat{z}^t \), which is exogenously given to the industry in the beginning of period \( t \). The optimal technology is \textit{ex ante} unknown to the firms and is not necessarily the same as \( \hat{z}^t \).

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7 It is shown in Section 5 that the dynamic model presented in this paper does predict the same relationships along the steady state when the industry is subject to persistent technology shocks.
Central to the model is the view that the firms engage in search for the optimal technology over time, but with limited foresight. What makes this “perennial” search non-trivial is the stochastic nature of the production environment – i.e., the technology which was optimal in one period is not necessarily optimal in the next period. This is captured by allowing the optimal technology, \( \hat{x}_t \), to vary from one period to the next in a systematic manner. The mechanism that guides this shift dynamic is described next.

### 2.2.1. Turbulence in the Technological Environment

Consider a binary vector, \( x \in \{0,1\}^N \). Define \( \delta(x, l) \subseteq \{0,1\}^N \) as the set of points that are exactly Hamming distance \( l \) from \( x \). The set of points that are within Hamming distance \( l \) of \( x \) is then defined as

\[
\Delta(x, l) \equiv \bigcup_{i=0}^{l} \delta(x, i).
\]  

(11)

The following rule governs the shift dynamic of the optimal technology:

\[
\hat{x}_t = \begin{cases} 
\hat{x}_t' & \text{with probability } \gamma \\
\hat{x}_{t-1} & \text{with probability } 1 - \gamma 
\end{cases}
\]

where \( \hat{x}_t' \in \Delta(\hat{x}_{t-1}, g) \) and \( \gamma \) and \( g \) are constant over all \( t \). Hence, with probability \( \gamma \) the optimal technology shifts to a new one within \( g \) Hamming distance from the current technology, \( \hat{x}_{t-1} \), while with probability \( 1 - \gamma \) it remains unchanged at \( \hat{x}_{t-1} \). The volatility of the technological environment is then captured by \( \gamma \) and \( g \), where \( \gamma \) is the rate and \( g \) is the maximum magnitude of changes in technological environment. For the computational experiments reported here, \( \hat{x}_t' \) is chosen from \( \Delta(\hat{x}_{t-1}, g) \) according to the uniform distribution.

The change in technological environment is assumed to take place in the beginning of each period before firms make any decisions. While the firms do not know what the optimal technology is for the new environment, they are assumed to get accurate signals of their own marginal costs based on the new environment when making their decisions to enter or to perform R&D.\(^8\) This is clearly a strong assumption. A preferred approach would have been to explicitly model the process of learning about the new technological environment; it is for analytical simplicity that I abstract away from this process.

### 2.2.2. Multi-Stage Decision Structure

The technological environment, \( \hat{x}_t \), is defined at the start of each period before firms engage in their decision making. Each period consists of four decision stages – see Figure 1. Denote by \( S_{t-1} \) the set of surviving firms from \( t-1 \), where \( S^0 = \emptyset \). The set of surviving firms includes those firms which were active in \( t-1 \) in that their outputs were strictly positive as well as those firms which were inactive with their plants shut down during the previous period. The inactive firms in \( t-1 \) survive to \( t \) if and only if they have sufficient net wealth to cover their fixed costs in \( t-1 \). Each firm \( i \in S_{t-1} \) possesses a production technology, \( \hat{x}_{t-1}^i \), carried over from \( t-1 \), which gave rise to

\(^8\) It should be noted that more than one optimal technology can yield the same marginal cost for the firm given its technology.
its marginal cost of \( c_{t-1} \) as defined in equation (4). It also has the current net wealth of \( w_t \) it carries over from \( t - 1 \).

Let \( R^t \) denote a finite set of potential entrants who contemplate entering the industry in the beginning of \( t \). I assume that the size of the potential entrants pool is fixed and constant at \( r \) throughout the entire horizon. I also assume that this pool of \( r \) potential entrants is renewed fresh each period. Each potential entrant \( k \) in \( R^t \) is endowed with a technology, \( z_k \), randomly chosen from \( \{0,1\}^N \) according to the uniform distribution. In addition, each potential entrant has a fixed start-up wealth with which it enters the market.

The definitions of the set notations introduced in this section and used throughout the paper are summarized in Table 1.

**Stage 1: Entry Decisions**

In stage 1 of each period, the potential entrants in \( R^t \) first make their decisions to enter. We will denote by \( b \) the fixed “start-up” wealth common to all potential entrants. The start-up wealth, \( b \), may be viewed as a firm’s available funds that remain after paying for the one-time set-up cost of entry. For example, if one wishes to consider a case where a firm has zero fund available, but must incur a positive entry cost, it would be natural to consider \( b \) as having a negative value.

It is important to specify what a potential entrant knows as it makes the entry decision. A potential entrant \( j \) knows its own marginal cost, \( c_j \), which is based on its technology, \( z_j \), and the new environment, \( \hat{z}^t \): It is not that the potential entrant \( j \) knows the content of \( \hat{z}^t \) (the optimal method for each activity), but only that it gets an accurate signal on \( c_j \) (which is determined by \( \hat{z}^t \)). The potential entrant also has observations on the market price and the incumbent firms’ outputs from \( t - 1 \), i.e., \( \bar{P}^{t-1} \) and \( q_i^{t-1} \) \( \forall i \in S^{t-1} \). Given these observations and the fact that \( q_i = s[\bar{P} - c_i] \) from equation (8), \( j \) can infer \( c_i^{t-1} \) for all \( i \in S^{t-1} \). While the surviving incumbent’s marginal cost in \( t \) may be different from that in \( t - 1 \), I assume that the potential entrant takes \( c_i^{t-1} \) to stay fixed for lack of information on \( \hat{z}^t \). The potential entrant \( j \) then uses \( c_j \) and \( \{c_i^{t-1} \}_{i \in S^{t-1}} \) in computing the post-entry profit expected in \( t \). Table 2 summarizes the decision environment for the potential entrant as well as for the incumbents.

Given the above information, the entry rule for a potential entrant takes the simple form that it will be attracted to enter the industry if and only if it perceives its post-entry net wealth in period \( t \) to be strictly positive. The entry decision then depends on the profit that it expects to earn in \( t \) following entry, which is assumed to be the static Cournot equilibrium profit based on the marginal costs of the active firms from \( t - 1 \) and itself as the only new entrant in the market. That each potential entrant assumes itself to be the only firm to enter is clearly a strong assumption. Nevertheless, this assumption is made for two reasons. First, it has the virtue of simplicity. Second, Camerer and Lovallo (1999) provide support for this assumption by showing in an experimental setting of business entry that most subjects who enter tend to do so with overconfidence and excessive optimism. Furthermore, they find: “Excess entry is much larger when subjects volunteered to participate knowing that payoffs would depend on skill. These self-selected subjects seem to neglect the fact that they are competing with a reference group of subjects who all think they are skilled too.”

The decision rule of a potential entrant \( k \in R^t \) is then:
\[
\begin{cases}
  \text{Enter,} & \text{if and only if } \pi_k^e(z_k^e) + b > W_j; \\
  \text{Stay out,} & \text{otherwise}
\end{cases}
\] (12)

where \( \pi_k^e \) is the static Cournot equilibrium profit the entrant expects to make in the period of its entry and \( W_j \) is the threshold level of wealth for a firm's survival (common to all firms).

Once every potential entrant in \( R_t \) makes its entry decision on the basis of the above criterion, the resulting set of actual entrants, \( E^t \subseteq R^t \), contains only those firms with sufficiently efficient technologies to guarantee some threshold level of profits given its belief about the market structure and the technological environment. Denote by \( M_t^e \) the set of firms ready to compete in the industry: \( M_t^e \equiv S_{t-1} \cup E^t \). At the end of stage 1 of period \( t \), we have a well-defined set of competing firms, \( M_t^e \), with their current net wealth, \( \{w_i^{t-1}\}_{i \in M_t^e} \) and their technologies, \( z_i^{t-1} \) for all \( i \in S_{t-1} \) and \( z_j^t \) for all \( j \in E^t \).

**Stage 2: R&D Decisions**

In stage 2, the surviving incumbents from \( t = 1, S_{t-1} \), engage in R&D to improve the efficiency of their existing technologies. Given that the entrants in \( E^t \) entered with new technologies, they do not engage in R&D in \( t \). In addition, only those firms with sufficient wealth to cover the R&D expenditure engage in R&D. I will denote by \( l_i^t \) the R&D expenditure incurred by firm \( i \) in \( t \).

The R&D process transforms the incumbent’s technology from \( z_i^{t-1} \) to \( z_i^t \), where \( z_i^t = z_i^{t-1} \) if either no R&D is performed in \( t \) or R&D is performed but its outcome is not adopted. The modeling of this transformation process is described separately and in full detail in Section 2.2.3.

**Stage 3: Output Decisions and Market Competition**

Given the R&D decisions made in stage 2 by the firms in \( S_{t-1} \), all firms in \( M_t^e \) now have the updated technologies \( \{z_i^t\}_{i \in M_t^e} \). With the updated technologies, the firms engage in Cournot competition in the market, where we represent the outcome with the Cournot equilibrium defined in Section 2.1.2.

Recall that the equilibrium in Section 2.1.2 was defined for \( m^t \) firms under the assumption that all \( m^t \) firms produce positive quantities. In actuality, given the asymmetric costs, there is no reason to think that all firms in \( M_t^e \) will produce positive quantities in equilibrium. Some relatively inefficient firms may shut down their plants and stay inactive (but still pay the fixed cost). What we need is a mechanism for identifying the set of active firms out of \( M_t^e \) such that the Cournot equilibrium among these firms will indeed entail positive quantities only. This is done in the following sequence of steps. Starting from the initial set of active firms, compute the equilibrium outputs for each firm. If the outputs for one or more firms are negative, then de-activate the least efficient firm from the set of currently active firms, i.e., set \( q_i^t = 0 \) where \( i \) is the least efficient firm. Re-define the set of active firms (as the previous set of active firms minus the de-activated firms) and re-compute the equilibrium outputs. Repeat the procedure until all active firms are producing non-negative outputs. Each inactive firm produces zero output and incurs the economic loss equivalent to its fixed cost. Each active firm produces its equilibrium output and earns the corresponding profit. We then have \( \pi_i^t \) for all \( i \in M_t^e \).
Stage 4: Exit Decisions

Given the single-period profits or losses made in stage 3 of the game, the firms in $M^t$ consider exiting the industry in the final stage. Each firm’s net wealth is first updated on the basis of the profits (or losses) made in stage 3 as well as the R&D expenditure incurred in stage 2:

$$w^t_i = w^{t-1}_i + \pi^t_i - l^t_i$$

(13)

where $l^t_i$ is the firm’s R&D expenditure made in stage 2. The exit decision rule for each firm is:

$$\begin{cases} 
\text{Stay in, if and only if} & w^t_i \geq W; \\
\text{Exit, otherwise}; 
\end{cases}$$

(14)

where $W$ is the previously-defined threshold level of net wealth such that all firms with current net wealth below $W$ exit the market. Define $L^t$ as the set of firms which exit the market in $t$. Once the exit decisions are made by all firms in $M^t$, the set of surviving firms from period $t$ is defined as:

$$S^t \equiv \{ \text{all } i \in M^t | w^t_i \geq W \}.$$  

(15)

The set of surviving firms, $S^t$, their current technologies, $\{ z^t_i \}_{i \in S^t}$, and their current net wealth, $\{ w^t_i \}_{i \in S^t}$, are then passed on to $t + 1$ as state variables.

2.2.3. Making the Process of R&D Endogenous

The process of R&D is made endogenous in this model. This process corresponds to the stage-2 process of transforming $z^t_{i-1}$ to $z^t_i$ as described in Section 2.2.2. I model the R&D-related decisions as being driven by a set of choice probabilities that evolve over time on the basis of a reinforcement learning mechanism. If a firm decides to pursue R&D, it can do so through either innovation or imitation. The size of R&D expenditure depends on which of the two modes a given firm chooses: innovation costs a fixed amount of $K_{IN}$ while imitation costs $K_{IM}$. Hence, the sufficient condition for a firm to engage in R&D is to have enough net wealth to cover the maximum R&D expense:

$$w^{t-1}_i \geq \max \{K_{IN}, K_{IM}\}.$$  

(16)

In the computational experiments reported here I assume $K_{IN} > K_{IM}$.

Figure 2 illustrates the various stages of the R&D process. The crucial part of this model is how the various components of the R&D decision are carried out. First, each firm $i$ has two probabilities, $\alpha^t_i$ and $\beta^t_i$, which evolve over time via a reinforcement learning mechanism. Each period, firm $i$ chooses to pursue R&D with probability $\alpha^t_i$ and not to pursue R&D with probability $1 - \alpha^t_i$. If the firm chooses not to pursue R&D, it simply keeps the old technology and, hence, $z^t_i = z^{t-1}_i$. However, if the firm chooses to pursue R&D, then it has a probability $\beta^t_i$ with which it chooses to “innovate” and $1 - \beta^t_i$ with which it chooses to “imitate.” (As mentioned, both $\alpha^t_i$ and $\beta^t_i$ are endogenous – how they are updated from one period to the next is discussed below.)

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9 It does not matter whether R&D expenditure is subtracted from the net wealth in stage 2 or in stage 4. It is a sunk cost by the time market competition starts and, as such, it has no impact on the firm’s output decision in stage 3.
Innovation occurs when the firm considers changing the method (i.e., flipping the bit) in one randomly chosen activity. Imitation occurs when the firm (i) picks another firm (j) from a subset of $S_{t-1}^i$ and considers copying the method employed by j in one randomly chosen activity while retaining his (i’s) current methods in all other activities. [Hence, the imitating firm is capable of copying only a small part of the entire technology.10]

Only those surviving firms which were profitable in $t-1$, i.e., $\pi_{t-1}^i > 0$, are considered as potential targets for imitation. Let $S_{t-1}^+$ denote the set of these profitable firms, where $S_{t-1}^+ \subseteq S_{t-1}$. The choice of a firm to imitate is made probabilistically using the “roulette wheel” algorithm. To be specific, the probability of firm $i \in S_{t-1}$ observing a firm $j \in S_{t-1}^+$ is denoted $p_{ij}^t$ and is defined as:

$$p_{ij}^t = \frac{\pi_{t-1}^j}{\sum_{k \in S_{t-1}^+, k \neq i} \pi_{t-1}^k} \quad (17)$$

such that $\sum_{j \in S_{t-1}^+, j \neq i} p_{ij}^t = 1 \forall i \in S_{t-1}$. Hence, the more profitable firm is more likely to be imitated.

Let $\tilde{x}_{ij}^t$ denote firm $i$’s vector of experimental methods (i.e., a technology considered for potential adoption) obtained through innovation or through imitation. The adoption decision rule is:

$$\tilde{z}_{ij}^t = \begin{cases} \tilde{x}_{ij}^t, & \text{if and only if } c_i(\tilde{z}_{ij}^t, \tilde{x}_{ij}^t) < c_i(\tilde{z}_{ij}^{t-1}, \tilde{x}_{ij}^t); \\ \tilde{z}_{ij}^{t-1}, & \text{otherwise.} \end{cases} \quad (18)$$

Firm $i$ adopts the proposed technology if and only if it lowers the marginal cost below the level attained with the current technology the firm carries over from the previous period.11 Hence, adoption happens when the Hamming distance to the optimal technology is lower with the proposed technology than with the current technology. Notice that this condition is equivalent to a condition on the firm profitability. When an incumbent firm takes all other incumbent firms’ marginal costs as given, the only way that its profit is going to improve is if its marginal cost is reduced as the result of its R&D.

Note that firm $i$’s R&D expenditure in period $t$ depends on the type of R&D activity it pursues:

$$l_i^t = \begin{cases} 0, & \text{if no R&D was pursued;} \\ K_{IN}, & \text{if R&D was pursued and innovation was chosen;} \\ K_{IM}, & \text{if R&D was pursued and imitation was chosen.} \end{cases} \quad (19)$$

Let us get back to the choice probabilities, $\alpha_i^t$ and $\beta_i^t$. Both probabilities are endogenous and specific to each firm. Specifically, they are adjusted over time by individual firms according to a reinforcement learning rule. I adopt a version of the Experience-Weighted Attraction (EWA) learning rule as described in Camerer and Ho (1999). Under this rule, a firm has a numerical

---

10 This is one aspect of the cognitive limitation assumed in this research. An issue that can be investigated in the future is to relax this assumption and examine the impact that a firm’s cognitive capacity has on the various outcomes at the firm and industry level. This is not pursued here.

11 I assume that the evaluation of the technology by a firm in terms of its production efficiency (as represented by the level of its marginal cost) is done with perfect accuracy. While this assumption is clearly unrealistic, it is made to avoid overloading the model.
attraction for each possible course of action. The learning rule specifies how attractions are updated by the firm’s experience and how the probabilities of choosing different courses of action depend on these attractions. The main feature is that a positive outcome realized from a course of action reinforces the likelihood of that same action being chosen again.

Formally, the choice probabilities, $\alpha^t_i$ and $\beta^t_i$, are determined by the attraction measures, $(A^t_i, \bar{A}^t_i)$ and $(B^t_i, \bar{B}^t_i)$, as follows:

$$\alpha^t_i = \frac{A^t_i}{A^t_i + \bar{A}^t_i}; \quad \beta^t_i = \frac{B^t_i}{B^t_i + \bar{B}^t_i},$$

where $A^t_i$ is the attraction for R&D and $\bar{A}^t_i$ is the attraction for No R&D, while $B^t_i$ is the attraction for Innovation and $\bar{B}^t_i$ is the attraction for Imitation.

At the end of each period, $\alpha^t_i$ and $\beta^t_i$ are adjusted on the basis of the changing values for these attraction measures. Table 3 shows the adjustment dynamics of these attractions for the entire set of possible cases. According to this rule, $A^t_i$ is raised by a unit when R&D (either through innovation or imitation) was productive and the generated idea was adopted. Alternatively, $\bar{A}^t_i$ is raised by a unit when R&D was unproductive and the generated idea was discarded.

In terms of the choice between innovation and imitation, $B^t_i$ is raised by a unit if R&D via innovation was performed and the generated idea was adopted or if R&D via imitation was performed and the generated idea was discarded. Hence, the attraction for innovation can increase if either innovation was productive or imitation was unproductive. Conversely, $\bar{B}^t_i$ is raised by a unit if R&D via imitation generated an idea which was adopted – i.e., imitation was productive – or R&D via innovation generated an idea which was discarded – i.e., innovation was unproductive. If no R&D was performed, all attractions remain unchanged.

Finally, all new entrants in $E^t$ are endowed with the initial attractions that make them indifferent to the available options at the time of their entry. Specifically, I assume that $A^t_i = \bar{A}^t_i = 10$ and $B^t_i = \bar{B}^t_i = 10$ for new entrants such that $\alpha^t_i = \beta^t_i = 0.5$ for all $i$ – i.e., it has equal probabilities of choosing between R&D and No R&D as well as between innovation and imitation. Of course, these attractions will eventually diverge from one another as the firms go through different market experiences as the result of their R&D decisions made over time.

3. EXPERIMENTAL DESIGN

The computational approach taken in this study allows us to perform controlled experiments by “creating and growing” an industry in silico. The procedure starts with an empty industry – i.e., $S^0 = \emptyset$. The “birth” of the industry is signified by the initial group of firms who enter the industry in $t = 1$. The market competition following their entry determines the set of surviving incumbents who, along with a fresh set of potential entrants, undergo the multi-stage decision process described in Section 2. The dynamic interaction between the turbulent technological environment and the multi-agent decision process drives the growth and development of the industry in the long run.

12 The source code for the computational experiments was written in C++ and the simulation outputs were analyzed and visualized using Mathematica 7.0. The source code is available upon request from the author.
In this section, I specify the parameter values for a baseline case, and describe the various parameter configurations considered in the study as a whole. I also describe the endogenous variables that are tracked for analyses and provide a visual display of the growth and development path taken by an industry characterized by the baseline parameter configuration. The baseline provides a useful benchmark for the comparative dynamics analyses performed in the following sections.

3.1. Design of Computational Experiments

A particular industry is characterized by the set of parameters specified in the model. The values of the parameters used in this study, including those for the baseline simulation, are provided in Table 4.

The production process is specified to have 96 separate tasks \(N = 96\), where the method chosen for each task is represented by a single bit. This implies that there are \(2^{96} (\approx 8 \times 10^{28})\) different combinations of methods for the complete production process. In each period, there are exactly 40 potential entrants who consider entering the industry, where a new firm enters with a start-up wealth \(b\) of zero. An incumbent firm will exit the industry if his net wealth falls below the threshold level \(W\) of zero. The demand intercept \(a\) is fixed at 300. With the exception of the analysis carried out in Section 6, the cost of innovation, \(\beta_i^0\), is fixed at 100, while the cost of imitation, \(K_{IM}\), is fixed at 50. In Section 6, we examine the impact of the costs of R&D by considering four distinct pairs of \(K_{IN}\) and \(K_{IM}\): \((K_{IN}, K_{IM}) \in \{(100, 50), (300, 150), (500, 250), (700, 350)\}\).

All initial attractions for R&D activities are such that the new entrants are indifferent between R&D and No R&D \((A^0_i = A^0_{Ni} = 10)\) as well as between Innovation and Imitation \((B^0_i = B^0_{Ni} = 10)\). The rate of change in the technological environment is set at \(\gamma = 0.1\). The maximum magnitude of a change in technological environment, \(r\), is held fixed at 8 – i.e., the Hamming distance between the optimal technologies at \(t - 1\) and at \(t\) cannot be more than 8 bits. The time horizon \(T\) is over 5,000 periods, where in period 1 the market starts out empty. The examination of the simulation outputs shows that the horizon of 5,000 periods is more than enough for an industry to achieve a steady-state for all parameter values considered in this research.

For the analyses of the baseline and the long-run steady-state, I focus on the impacts of the market size \(s\) and the fixed cost \(f\) on the industry dynamics. I consider four different values for the two parameters: \(s \in \{3, 4, 5, 6\}\) and \(f \in \{200, 300, 400, 500\}\).

Starting from an empty industry with the above configuration of parameters, I evolve the industry and trace its development by keeping track of the following endogenous variables:

- \(|E^t|\): number of firms that entered the industry in the beginning of \(t\)
- \(|L^t|\): number of firms that left the industry at the end of \(t\)
- \(|M^t|\): number of firms that were in the industry in \(t\) (including both active and inactive firms)
- \(|S^t|\): number of firms that survived at the end of \(t\) (= \(|M^t| - |L^t|\))
- \(P^t\): market price at which goods were traded in \(t\)
- \(\{c^t_{vi}\}\)\(\forall vi\): realized marginal costs of all firms in the industry in \(t\)
- \(\{q^t_{vi}\}\)\(\forall vi\): actual outputs of all firms in the industry in \(t\)
- \(\{\pi^t_{vi}\}\)\(\forall vi\): realized profits (or losses) of all firms in the industry in \(t\)
- \(\{age^t_{vi}\}\)\(\forall vi\): ages of all firms in the industry in \(t\)
• \{\alpha_t^i\}_{i \in M^t}: R&D intensities of all firms in the industry in \(t\)
• \{\beta_t^i\}_{i \in M^t}: innovation intensities of all firms in the industry in \(t\)
• \{I_t^i\}_{i \in M^t}: R&D spending of all firms in the industry in \(t\) (\(I_t^i = 0\) if a firm did not perform any R&D; \(I_t^i = K_{IM}\) if firm \(i\) performed imitation.)

Using the above variables, I construct an additional group of endogenous variables that characterize the aggregate behavior of the firms in an industry. First, denote by \(Q^t\) and \(\Pi^t\) the aggregate output and the aggregate profit of all firms in period \(t\): 

\[Q^t = \sum_{i \in M^t} q_t^i\quad \text{and}\quad \Pi^t = \sum_{i \in M^t} \pi_t^i.\]

Note that both the size of the market \((s)\) and the fixed cost \((f)\) are likely to have significant influence on the number of firms that a given industry can sustain in the long run. Since the magnitude of firm turnovers must be viewed in relation to the size of the industry, I construct the rates of entry and exit, \(ER^t\) and \(XR^t\), which are, respectively, the number of new entrants and the number of exiting firms as the fractions of the total number of firms in period \(t\):

\[
ER^t = \frac{|E^t|}{|M^t|}\quad \text{and}\quad XR^t = \frac{|L^t|}{|M^t|}.
\]

The rate of firm survival in period \(t\) is then \(1 - XR^t\).

As a concentration measure, I use the Herfidal-Hirschmann Index, \(H^t\):

\[
H^t = \sum_{i \in M^t} \left(\frac{q_t^i}{Q^t} \cdot 100\right)^2.
\]

A novel aspect of the model is how technological heterogeneity leads to cost asymmetries among firms. To investigate the evolving technological heterogeneity within the industry, I introduce a measure of the “degree of technological diversity,” \(DIV^t\). It is defined as the ratio of the mean technological difference in the population of all firms to the maximum possible difference. To be specific, first note that the maximum difference between any two technologies is when their Hamming distance is \(N\). The mean Hamming distance, the numerator of the ratio, is computed as an average of the Hamming distances between all distinct pairs of firms within the population. Since the set of firms, \(M^t\), contains a total of \(|M^t|\) firms, the total number of distinct pairs that can be formed among them is: \(
\frac{1}{2} |M^t|(|M^t| - 1)\). The degree of technological diversity is then computed as:

\[
DIV^t = \frac{2}{N|M^t|(|M^t| - 1)} \sum_{i \neq j} D(z_i^t, z_j^t)
\]

The practical implication of the heterogeneity in firms’ technologies is the asymmetry it creates in terms of their production efficiency and the consequent market shares. Note that in each period \(t\), the market share of a firm \(i\) is defined as \(\frac{q_t^i}{Q^t}\). The inequality in market shares in \(t\) may then be represented by the Gini coefficient, \(G^t\), which is computed as:
To examine the aggregate intensity of the R&D activities, I look at the total R&D spending in the industry, $TRD^t$:

$$TRD^t = \sum_{\forall \in EM^t} I^t_\forall.$$

(25)

If a firm pursues R&D, it either innovates or imitates. The aggregate R&D expenditure, $\sum_{\forall \in EM^t} I^t_\forall$, in period $t$ then consists of the amount spent by the firms that innovate and the amount spent by those that imitate. [It should be noted that the inactive firms, producing zero output while paying the fixed cost, may still choose to pursue R&D and incur these expenses if they have sufficient net wealth.] Denote by $TCN^t$ the aggregate amount spent on innovation (rather than imitation) by all firms in period $t$. Let $NRD^t$ be the cost share of innovation in the aggregate R&D spending:

$$NRD^t = \frac{TCN^t}{TRD^t}.$$

(26)

$NRD^t$ measures the industry’s relative tendency to invest in innovation rather than in imitation.

For an aggregate measure of the industry’s production efficiency, I construct an industry marginal cost, $WM^t$, where

$$WM^t = \sum_{\forall \in EM^t} \left[ \left( \frac{q^t_\forall}{Q^t} \right) * c^t_\forall \right].$$

(27)

$WM^t$ is, hence, the weighted average of the individual firms’ marginal costs in period $t$, where the weights are the market shares of the firms in that period.

In order to evaluate the market power of the firms, I also construct an aggregate measure of firms’ price-cost margins, $PCM^t$, where

$$PCM^t = \sum_{\forall \in EM^t} \left[ \left( \frac{q^t_\forall}{Q^t} \right) * \left( \frac{P^t - c^t_\forall}{P^t} \right) \right].$$

(28)

$PCM^t$ is the weighted average of the individual firms’ price-cost margins in period $t$, where the weights are the market shares of the firms.

For a measure of consumer welfare, I compute the consumer surplus as the usual triangular area under the demand curve above the market price:

$$CS^t = \frac{1}{2} (a - P^t)Q^t,$$

(29)

where $P^t$ and $Q^t$ are the realized price and aggregate output in period $t$. 

$$G^t = \frac{2 \sum_{i=1}^{[M^t]} \left( i \cdot \frac{q^t_i}{Q^t} \right)}{|M^t|} - \frac{|M^t| + 1}{|M^t|}.$$ 

(24)
Finally, the total surplus that captures the overall social welfare is computed as the sum of consumer surplus and the aggregate profit:

\[ TS^t = CS^t + II^t. \]  

(30)

3.2. The Baseline: Generating the Proto-History

The first step in my analysis is to examine the evolving structure of a typical industry as characterized by the baseline parameter values indicated in Table 4. Since the market demand is fixed for all \( t \), any shift in the firms’ decision environment is solely due to the random shocks in the technological environment. The external technological shocks induce entry and exit of firms by directly influencing their current marginal costs, but they also give rise to adaptive R&D by firms in their search for the new technological optimum. After the initial transition period following its birth, the industry settles into a steady-state in which each endogenous variable representing the industry structure fluctuates around a constant mean with a finite variance.

I start by focusing on a single randomly chosen replication and observing the endogenous time paths of the three turnover variables over the 5,000 periods of the industry’s development from its birth to full maturity: (a) the number of entrants, \( |E^t| \); (b) the number of exiting firms, \( |L^t| \); and (c) the total number of firms, \( |M^t| \). The results are captured in Figure 3.

Note from Figure 3(a) the initial surge in the number of new entrants into the industry at its birth: The entire pool of potential entrants (40) jumps into the industry as it is newly born. This rush quickly slows down and the industry settles into a steady state where recurring waves of new entrants are observed over the horizon. The number of exits in figure (b) shows that the initial surge in entry is immediately followed by a large number of exits, implying that a large number of firms who initially entered the industry are soon forced out through a severe market competition – i.e., a “shakeout.” After the initial shakeout, the industry experiences steady out-flow of firms that accompanies the steady in-flow of firms exhibited in (a). Hence, we observe persistent positive levels of entry and exit.

The continual streams of entries and exits interact to produce the time series in figure (c) of the number of firms, \( |M^t| \), which includes both active and inactive firms. The time path shows that the number of firms moves with substantial volatility over time, though it moves around a steady moving average (≈ 41) after about \( t = 1,000 \). This suggests a positive correlation between the time series of \( |E^t| \) and \( |L^t| \). For the baseline run reported in Figure 3, the correlation between the numbers of entry and exit was 0.58, while that between the rates of entry and exit was 0.57. The positive correlation between the numbers (rates) of entries and exits holds for all other runs tried in this study. The co-movement of entry and exit rates is discussed in greater detail in Section 5.

The time paths captured in Figure 3 are representative of all replications performed in this study. For multiple replications using the same baseline parameter values but with fresh random numbers (which determine the starting technologies and periodic shocks), the distribution of the period-specific outcomes generated from the stochastic process tends to be time-invariant for \( t > 1,000 \). Figure 4 shows the time series outputs of the same variables as in Figure 3 as the mean over 500 independent replications: \( \left\{ \frac{1}{500} \sum_{k=1}^{500} X^t_k \right\}_{t=1}^{5000} \), where \( X^t_k \) is the value of the endogenous variable \( X \) at \( t \) in replication \( k \). As expected, the number of firms on average attains a stable level by \( t = 1,000 \).
Further support for the convergence to a steady state is provided in Figure 5 which shows the time series plots of other endogenous variables: a) market price \( (P) \); b) industry marginal cost \( (WMC) \); c) industry price-cost margin \( (PCM) \); d) aggregate profits \( (II) \). The time paths of interest always reach a steady state by \( t = 3,000 \) for all parameter configurations considered in this study. As such, when we examine the impact of industry-specific factors on the industry’s performance in Sections 5-6, the steady-state value of an endogenous variable will be computed as an average over the last two thousand periods between \( t = 3,001 \) and \( t = 5,000 \).13

Finally, Figure 6 shows the evolving degree of technological diversity \( (DIV) \) and the market share inequality \( (G) \) over time. The time series from a single replication is captured in (a) and (c), while those as an average over 500 independent replications are presented in (b) and (d), respectively for the two variables. The degree of technological diversity starts at about 0.5 in the beginning, but rapidly declines to approach the steady-state mean of about 0.45. The steady-state is clearly reached by \( t = 1,000 \). The market share inequality, as measured by the Gini coefficient, takes a severe dip during the first wave of shakeouts immediately following the birth of the industry; but it soon climbs back up to a stable level (\( \approx 0.38 \)) as the industry converges to a steady state.

4. SHAKEOUTS: LIMITED FORESIGHT, TECHNOLOGICAL SHOCKS, AND TRANSIENT INDUSTRY DYNAMICS

It is well-known that the market histories of many manufacturing and service industries in their infancy display what is commonly known as a “shakeout.” The phenomenon refers to the rapid rise in the number of producers at the opening of the new market, followed by a sharp decline (a shakeout), then an eventual convergence to a stable structure. The U.S. automobile industry in its early years offers one of the more striking examples. Using the year 1895 as the founding year for the industry, Smith (1968) traces the growth and development of the industry from 1895 to 1966. He conveys in the following passage the degree of initial optimism held by the individual producers but also the general concern for the extent of its excess:

By 1905, with the decade half over, 183 new companies had undertaken the building of pleasure cars (the term then used to designate passenger cars) and 93 companies had ceased production. … The situation was precarious with so many entering the business and sales so hard to make, and E. H. Cutler, president of the Knox Automobile Co. in Springfield, Massachusetts, was voicing the concern of many when he said: “There will be a danger from the attempt to manufacture and sell large quantities of machines that have not been fully tested; and there is a limit to the quantity that this or any other country can absorb, and we are inclined to advise conservatism in the planning of production.” No one heeded this warning, no one applied brakes, and by the end of the decade 531 companies had been formed (more than one a week for ten years) and 346 had gone out of business, most through bankruptcy. [Smith (1968), p.25]

To see this more systematically, I plot in Figure 7 the number of passenger car companies in the U.S. between 1895 and 1966 based on the data compiled by Smith (1968).14 Figure 8 plots both the

13 See Law and Kelton (2000) for a detailed discussion of how to identify the steady state in stochastic processes. Chapter 9 on “Output Data Analysis for a Single System” is particularly useful.

14 The original list compiled by Smith (1968) – referred to as “basic list A” – contains all “makes” of passenger cars produced commercially in the United States from 1895 to 1966 along with their years of production. The year of entry for a company is hence represented by the year production commenced,
number of entering companies and that of exiting companies over the same time period. While the shakeout pattern is unmistakable in Figure 7, the entry/exit data in Figure 8 also indicate that the entry and exit of firms tend to occur simultaneously – a pattern that is hard to explain using the standard textbook economic theory in which the existence of economic profits invites entry while the existence of economic losses promotes exit, but never both at the same time. This general pattern also implies a substantial degree of infant mortality, which is captured in Figure 9. It plots the proportion of all exiting firms (over 1895-1966) that, at the time of exit, were of age younger than or equal to AGE (indicated along the horizontal axis). It is significant that almost 90% of those who exited were 10 years of age or younger.

The body of scholarly work addressing the phenomenon of shakeout is not large. Gort and Klepper (1982) first offered a systematic study by tracing the market histories of 46 new products, identifying a distinct sequence of stages observed in the development of various industries from their birth to maturity. The findings were further elaborated upon by Klepper and Simons (1997, 2000a,b), and Klepper (2002). Carroll and Hannan (2000) has offered additional empirical evidence of the shakeout phenomenon in their comprehensive study of corporate demography.

There are two papers that offered stylized theoretical models capable of generating the shakeout patterns: Klepper and Graddy (1990) and Jovanovic and MacDonald (1994). Central to Klepper and Graddy’s (1990) model is a group of heterogeneous potential entrants, who differ from one another in terms of their costs and product qualities. They have perfect foresight and, as such, make their entry decisions based on the expected discounted profits. Given the heterogeneity in their costs and product qualities, only a small number of the potential entrants actually enter the industry. The incumbent firms are also heterogeneous in their costs and product qualities, where these differentials tend to persist because of the imperfection in the imitation activities of the firms. The shakeout pattern is generated through the randomness in the firms’ cost draws, the improvements of the cost positions through one-time imitation upon entry, and the eventual exits of those with cost positions that remain above the falling market price even after the initial improvement. Jovanovic and MacDonald (1994), using a similar conceptual framework, models the process of market competition as one in which one major invention is followed by a one-time refinement of the technology. The model is then estimated using data from the U.S. automobile tire industry.

It is notable that both Klepper and Graddy (1990) and Jovanovic and MacDonald (1994) were able to generate the shakeout pattern using purely analytical models. However, there are two reasons why I find these models less than satisfactory. First, both models focus strictly on generating the shakeout during the infant phase of an industry; there are no persistent entries and exits in the long run. Hence, the scope of analysis rendered by these models tends to be rather limited. Second, both of these models employ the standard assumption of a firm as the maximizer of expected profits with perfect foresight. As discussed earlier, the environment under study is one in which there exists a great deal of uncertainty, subject to external shocks to the technological environment, perpetual innovation, imperfect imitation, and constant threats of entry by outside firms. Given the inherent randomness in this process, it is difficult to accept, on conceptual grounds, the assumption of perfect rationality and foresight.

4.1. Shakeout in an Infant Industry

For purposes of evaluating the baseline model’s ability to simulate a real industry, I look at a case closely related to the automobile industry. Jovanovic and MacDonald (1994) provide a clean

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while the year of exit is represented by the year production ceased. The list was converted into a manageable dataset by Endrit Meta.
dataset on the shakeout phenomenon in the U.S. automobile tire industry. Using their data, I plot in Figure 10 the following: (a) the number of producers over 68 years from 1906 to 1973, (b) the wholesale price index over 61 years from 1913 to 1973, and (c) the industry output over 64 years from 1910 to 1973. As expected, the number of producers rises sharply in the beginning, reaching the maximum of 275 in 1922. It then declines sharply, eventually leveling off to a stable level. The wholesale price index declines over time, while the industry output rises steadily.

Given the empirical regularities presented in Figure 10, I now present the time series values of the relevant variables from a single sample run of my computational model, where the technological optimum is specified once in the beginning and stays fixed over the entire horizon – i.e., $\gamma = 0$. Other parameter values are set at the baseline levels indicated in Table 4. Assuming stable technological environment allows us to isolate the shakeout at the infancy of the industry, thus enabling clear identification of the forces that give rise to such a phenomenon.

Figure 11 shows the time paths of the three variables from a single typical replication: (a) the total number of firms ($M^t$); (b) the market price ($P^t$); and (c) the aggregate output of the industry ($Q^t$). These time paths are plotted only for the first 68 periods to facilitate comparisons with the empirical data plotted in Figure 10.

The qualitative similarities between Figure 10 and Figure 11 are striking. The number of firms in the computationally generated industry shoots up in the beginning when all potential entrants decide to enter. As many of these early entrants immediately exit the industry, the number of firms sharply drops down to a relatively stable level where it fluctuates around 28 firms per period. The market price starts out high in the beginning, but drops down gradually over time, while the aggregate output tends to rise over time. The behaviors of all three variables are consistent with those reported in Figure 10.

Figure 12 shows that the shakeout phenomenon captured in Figure 11 is not just restricted to the single replication. For a select group of endogenous variables, I plot their time series as averages over 500 independent replications (i.e., each replication using the same baseline parameter values but a fresh set of random numbers). In order to focus on the early stage where shakeout occurs, I plot the time series on a log-linear format. Again, the number of entrants is high at the birth of the industry and declines monotonically over time. The number of exits is also high and immediately follows the movement of entries, but it also tends to decline after the initial stage. The number of firms ($M^t$) in (c) displays the shakeout pattern as expected. The measure of industry concentration (HHI), $H^t$, dips in the beginning during the shakeout but rises gradually over time. The aggregate output ($Q^t$) increases and the market price ($P^t$) declines over time. Additionally, the industry marginal cost ($WMC^t$) declines monotonically, as shown in (g): Throughout the process of shakeout, the selective force of market competition pushes out the inefficient firms and permits the survival of only those with sufficiently low marginal costs. As new entrants come in with even lower marginal costs, the industry marginal cost continues to decline. While both the market price and the marginal costs decline over time, the rise in the industry price-cost margin ($PCM^t$), as shown in (h), indicates that the latter effect tends to dominate.

Finally, Figure 13 shows that the degree of technological diversity ($DIV^t$) and the market share inequality ($G^t$) both decline over time. In the context of our model, this is consistent with the monotonically declining industry marginal cost in Figure 12(g). As more efficient firms are selected

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15 Technological diversity declines monotonically throughout the entire horizon. The market share inequality, while also showing a general decline over time, displays a slight break in the earlier part of the horizon between $t = 30$ and $t = 200$. 

21
through market competition, these firms tend to have technologies that are closer to the (fixed) technological optimum and thus are closer to one another in the technology space; hence the uniform decline in technological diversity in (a). The technological convergence also implies that the market shares of the firms are increasingly equalized over time, as shown in (b). In the absence of external shocks to the technological environment, the degree of technological diversity and the market share inequality both converge to zero in the long run.

4.2. Technological Change and Recurrent Shakeouts

In Section 4.1, I focused on the shakeout in an infant industry by keeping the technological optimum fixed throughout the entire horizon. More generally, the technological environment is likely to be subject to recurring shocks. In this model, the persistence of firm entries and exits over time comes from the unexpected shifts in the technological environment surrounding the firms (which happens at the rate of \( \gamma \), where \( \gamma = 0.1 \) for the baseline case). To verify the relevance of this mechanism, I track the occurrences of technological shifts over the entire horizon. For a given technological shift that occurs in period \( \tau \), I define its “episode” as those consecutive periods following the shift before the next technological shift occurs in period \( \tau' \). The duration of the episode is then \((\tau' - \tau)\).

For the baseline run captured in Figure 3, there were 504 (completed) episodes of varying durations over the entire horizon of 5,000 periods. This number of episodes is as expected, given the rate of technological change at \( \gamma = 0.1 \).\(^{16}\) The durations of the episodes, however, are quite variable. Figure 14 shows how many times an episode of a given duration appeared over the time periods between \( t = 1,001 \) and 5,000 – the first 1,000 periods are excluded because of its transient nature. Over these 4,000 periods the longest episode had the duration of 66 (there was one such episode), while the shortest episode lasted only 1 period (there were 45 of them).

Figure 15 plots each episode in terms of its duration (along the horizontal axis) and its size (along the vertical axis) as measured by the total number of entries (left figure) and by the total number of exits (right figure) that occurred during the episode. As expected, they are highly correlated: An episode of longer duration tends to entail greater number of entries and exits.

To see the impact that technological shifts have on the turnover of firms in greater detail, I now ask, for each period over the horizon \( 1,001 \leq t \leq 5,000 \), how many periods have elapsed since the last technological shift. This allows me to examine the relationship between the rates of entry and exit and the elapsed time since a technological shift. Figure 16(a) and Figure 16(b) capture this information. On average, both rates tend to fall as the given period is further away from the last technological shift. [The correlations between the rates and the time since the last technological shock are noted on the corresponding figures.] The total number of firms [Figure 16(c)] is also negatively correlated with the time since the last technological shift, even though the industry concentration (HHI) in Figure 16(d) shows a very weak positive correlation.

These correlations indicate that the relationships between these turnover variables and the time since the last technological shock are similar in nature to those identified in the case of shakeouts in infant industries with no technological shocks. In the presence of recurrent technological shifts, one may then view the turnover dynamics as being a series of mini-shakeouts, in which the rates of entry and exit jump up immediately following a technological shift and then gradually fall down as the market adjusts to the new environment. The persistent series of entries and exits over time, as

\(^{16}\) With \( \gamma = 0.1 \), a new episode is expected every ten periods. Hence, a total of about 500 episodes should be expected over the horizon of 5,000 periods.
well as the positive correlation between the time series of the corresponding rates, are then a natural consequence of these repeated shakeouts following technological shifts.

The recurring shakeouts also display noticeable patterns in other endogenous variables; these patterns are consistent with those identified in the infant industries. Figure 17(a) shows that the market price is significantly negatively correlated with the time since the last technological shift. As previously shown, this is due to the drop in the marginal costs resulting from the market selection: The industry marginal cost is also negatively correlated with the time since the last technological shift – see Figure 17(b). Industry price-cost margin is positively correlated with the shift, while the degree of technological diversity is negatively (though weakly) correlated.

5. INDUSTRY DYNAMICS IN THE STEADY STATE: BETWEEN-INDUSTRY VARIATIONS

The baseline analysis in Section 3 indicated that, even with the persistent shocks to the firms’ technological environment, the industry eventually reaches a steady state in which a typical endogenous variable fluctuates around a constant mean. However, the shakeout analysis in Section 4 also indicated that even within the steady state there are systematic patterns across time as the result of external shocks to the technological environment. In this section, I pursue two objectives: 1) to identify and explain the temporal patterns at the aggregate industry level that exist along the steady-state path; and 2) to perform a comparative dynamics analysis of the steady state with respect to the two parameters, the size of the market ($s$) and the fixed cost ($f$), which represent the factors specific to an industry.

The empirical relevance of the comparative study of steady-states presented in this section can be seen in the observed variation in entry and exit patterns across industries as reported in Dunne, Roberts, and Samuelson (1988) [DRS (1988) from here on]. I plot in Figure 18 the summary measures of the entry and exit patterns across industries obtained from DRS (1988). Specifically, DRS (1988) average the rate of entry and the rate of exit over the four time-period observations for the four-digit SIC industries from their sample.17 Grouping the four-digit industries into twenty two-digit sectors, they then report the distributions of these average rates for each sector. In Figure 18 I plot the sector-averages of these rates for all twenty sectors in their study, where the horizontal coordinate and the vertical coordinate of a given point (sector) capture the average rate of entry and the average rate of exit, respectively, for the sector.

The way these points are dispersed along the diagonal makes it quite clear that the rate of entry and the rate of exit are positively correlated (at 0.732) across all sectors: A sector with a higher (lower) than average rate of entry also tends to have a higher (lower) than average rate of exit. For instance, the Instruments sector (SIC 38) has the highest rate of entry (0.603) as well as the highest rate of exit (0.468) in this sample, while the Tobacco sector (SIC 21) has the lowest rate of entry (0.205) and the lowest rate of exit (0.223). Hence, the industries vary in terms of the severity of turnovers. The comparative dynamics analysis carried out in this section will show that our model indeed generates this type of diversity along the steady state of the stochastic process and enable us to identify the causal relationships between the underlying parameters and these variations.

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5.1. Defining the Steady State

I start by providing a precise description of the process through which I study the steady state. For a given parameter configuration, I perform 500 independent replications, using fresh sequence of random numbers for each replication. The time series values of the endogenous variables are collected for the last 2,000 periods from \( t = 3,001 \) to \( t = 5,000 \). These time series characterize the steady-state paths of these endogenous variables.

Suppose a given replication \( k \) generates time series values for an endogenous variable \( X \) as \( \{X^t_k\}_{t=1}^{5,000} \), where \( X^t_k \) is the value of \( X \) in period \( t \) from replication \( k \). The steady-state mean of \( X \) for the given replication \( k \) is denoted \( \bar{X}_k \), where \( \bar{X}_k = \frac{1}{2,000} \sum_{t=3,001}^{5,000} X^t_k \). For each endogenous variable, \( X \), there will be 500 steady-state means (from 500 independent replications), \( \{\bar{X}_k\}_{k=1}^{500} \). The mean and the standard deviation of these 500 steady-state means, generated under the baseline parameter configuration, are reported in Table 5. It is notable that the aggregate profit, \( \Pi^t \), is negative on average. This reflects the significant economic losses taken by the inactive firms who are not currently in operation (though still paying for the fixed costs). These firms linger on as long as their accumulated net wealth exceeds the threshold level that reflects the opportunity cost of their resources.

When performing the comparative dynamics analysis, the average behavior of the industry with respect to a given endogenous variable \( X \) is represented by averaging \( \bar{X}_k \) over all replications: \( \bar{X} = \frac{1}{500} \sum_{k=1}^{500} \bar{X}_k \).

5.2. Temporal Patterns along the Steady State within an Industry

Recall from Section 4 that the long-run dynamics of the firms can be understood as recurrent shake-outs in the presence of persistent changes in the technological environment. A logical implication of that perspective was the correlated movement of firms in and out of the market, which also had implications for the performance of firms and the industry over time. To confirm the presence and the significance of the relationships between the endogenous variables, I report in Table 6 the correlations between the steady-state time series of all relevant pairs of the endogenous variables, \( X \) and \( Y \), over the steady state. The value in each cell is the average of these correlations from 500 independent replications: \( \bar{\rho}_{XY} = \frac{1}{500} \sum_{k=1}^{500} \text{Corr} [\{X^t_k\}_{k=1}^{5,000}, \{Y^t_k\}_{k=1}^{5,000}] \).

To begin, the rates of entry and exit, \( ER^t \) and \( XR^t \), are positively correlated at 0.321. Although not included in the table, the number of entries, \( |E^t| \), and the number of exits, \( |L^t| \), are also positively correlated with the mean value of 0.371.

Property 1: The rate of entry and the rate of exit are positively correlated over time: When the rate of entry is high (low), the rate of exit is also high (low).

This result is consistent with the third stylized fact mentioned in Introduction. Of course, this stylized fact is at odds with the standard textbook view that the presence of economic profits induces entry and that of economic losses induces exit (which implies a negative correlation between the rate of entry and the rate of exit over time). Commenting on the discrepancy between the accepted view and the empirical findings, Geroski (1995) observes: "This view is difficult to reconcile with the fact that a typical three digit industry in UK gained an average of 50 new firms per year over the period 1974-1979 and lost an average of 38, experiencing a net entry rate of just
over 1% and a negative net penetration rate of -0.42%.” He also finds from the Canadian data during the 1970s that the rate of entry and the rate of exit averaged 5% and 6.5%, respectively, resulting in a net entry rate of -1.5%. These findings indicate that the arrival of a large number of new firms tends to occur together with the departure of a large number of older firms, hence, implying a positive correlation between the rates.

The co-movement of the two rates suggests that the rate of firm turnover can be described by either the rate of entry or the rate of exit (or even by the sum of the two rates as specified in some of the past empirical works). A later result on comparative dynamics indicates that the two rates are positively correlated across different industries as well. Hence, I will simply use the rate of entry to capture the rate of turnover.

As we saw in Section 4, the positive correlation between the time series of the rate of entry and the rate of exit can be understood in the context of “recurrent shakeouts” in the presence of external technological shocks. The “recurrent shake-out” perspective also implies that the price must be positively correlated with both the entry and exit rates of firms, as it is when the technological environment shifts that the incumbent firms using the old technology will suddenly find themselves inadequately prepared for the new environment. The sudden drop in the average level of efficiency (and the corresponding rise in the firms’ marginal costs) leads to an increase in the market price. As firms adapt to the new environment and improve their production efficiency, the market price gradually declines while at the same time the entry and exit of firms slow down. Thus, the price, \( P^t \), is positively correlated with the rates of entry and exit. Indeed, the mean correlations are 0.318 and 0.208, respectively.

The aggregate profit, \( \Pi^t \), is negatively correlated with the entry and exit of firms. This is again due to the external shock in the technological environment adversely affecting the firms’ efficiency initially; the endogenous R&D raising it back up as firms adapt themselves to the new environment. As expected, the aggregate profit is also negatively correlated with the total number of firms in the industry.

Similar to the aggregate profit, the industry price-cost margin, \( PCM^t \), is negatively correlated with both entry and exit, as well as with the total number of firms. It is interesting to note, however, that it is negatively correlated with the market price, \( P^t \). The shake-out perspective provides an explanation. Note that the market price is driven by the level of firms’ efficiency: A high price reflects a high marginal cost (low efficiency), while a low price reflects a low marginal cost (high efficiency). Although the market price and the firms’ marginal costs tend to move in the same direction, the negative relationship between \( P^t \) and \( PCM^t \) suggests that a given change in the marginal cost is likely to have a less than equal effect on the market price.

The relationships between the industry marginal cost, \( WMC^t \), and various endogenous variables as reported in Table 6 are fully consistent with the shake-out-based explanations provided above. The industry marginal cost, \( WMC^t \), is positively correlated with entry, exit, and the number of firms. It also displays a strongly positive correlation (0.923) with the market price as expected, but a strongly negative correlation (-0.814) with the price-cost margin.

The concentration measure, \( H^t \), is negatively correlated with the number of operating firms (-0.565), while positively correlated with both the aggregate profit (0.799) and the price-cost margin (0.895): a period of higher concentration is also likely to show higher aggregate profits and a higher price-cost margin.
Property 2: The industry concentration is negatively correlated with the industry marginal cost over time, while it is positively correlated with the industry price-cost margin over time.

The positive correlation between the time series of industry concentration and price-cost margin would be logical in the context of oligopoly theory in which market concentration tends to raise the market power and the equilibrium firm profits. However, it is important to note that the underlying mechanism here relies less on the market power argument, but more on the efficiency effect. The external shock to the environment temporarily reduces the firms’ profits through an increase in their marginal costs (i.e., loss of efficiency), while it promotes turnovers that reduce the degree of concentration. It is the adaptive moves by the firms through their R&D that reverse these changes gradually until the next technological shock. The negative correlation (-0.475) between $H^t$ and $WMC^t$ is consistent with this explanation.

The variable capturing the R&D intensity within the industry, $TRD^t$, is positively correlated with the total number of firms, $|M^t|$, while negatively correlated with the aggregate profit, $II^t$, and the price-cost margin, $PCM^t$. The innovation’s share of the R&D cost, $NRD^t$, does not show any noticeable correlation with other variables, though it is positively correlated with the aggregate R&D spending ($TRD^t$).

The negative relationships between the performance measures (i.e., aggregate profits and the price-cost margin) and the R&D spending are due to the heightened R&D activities during the periods immediately following the external shock to the technological environment. A sudden change in the technological environment, while undermining the profitability of the previously well-adapted incumbents, also offers a fresh set of opportunities for the incumbents and the potential entrants alike.

The technological diversity ($DIV^t$) is positively correlated with the rates of entry and exit, as well as with the total number of firms. The external shocks to the technological environment bring about waves of entry and exit; the new entrants come in with a wide variety of new technologies. As the industry adapts to the new environment, the selective force of the market competition that weeds out the inefficient firms then leads to increased concentration and a reduction in the variety of surviving technologies; hence the negative correlation (-0.356) between $DIV^t$ and $H^t$. It is also notable that the technological diversity is positively correlated with the aggregate R&D (0.399).

Finally, the Gini coefficient, which measures the inequality in the market shares of the firms, shows a mild positive correlation with the entry/exit of firms, while a strongly positive correlation (0.787) with the number of firms.\(^\text{18}\) This indicates a more unequal market share distribution during those periods when the firms are actively moving in and out of the market. As such, it is also positively correlated with technological diversity (0.591). As the firms adapt to the new environment and the industry becomes more concentrated, their technologies (and their marginal costs) tend to converge, thus leading to a more equal distribution of market shares within the industry.

5.3. Between-Industry Variations in Steady States

While the non-equilibrium dynamics of the kind described above are common to all industries, the extent to which they affect the structure and performance of the industry along the steady-state path

\(^{18}\) The Gini coefficient is computed using the market shares of all firms that were in the industry in a given period, $t$. Hence, it includes all firms in $M^t$, which is defined after the entry stage but before the exit stage.
can differ from industry to industry. In this section, I compare the steady states of industries with different characteristics and explore the causal factors behind the observed heterogeneity.

There are four main parameters in the model that define the characteristics of an industry: the size of the market \(s\), the size of the fixed cost \(f\), the rate of change in the technological environment \(\gamma\), and the maximum magnitude of change in the technological environment \(g\). The latter two parameters, \(\gamma\) and \(g\), describe the turbulent nature of the external technological environment. The shocks to the technological environment are assumed exogenous to the industry and thus have an external source. Because the impacts of \(\gamma\) and \(g\) on the adaptive dynamics of the firms are rather intuitive and straightforward, I restrict the comparative dynamics analysis to \(s\) and \(f\) for expositional simplicity.

As described earlier, the comparative analysis focuses on the steady-state means of the endogenous variables averaged over 500 independent replications for each parameter configuration that represents a particular industry. This means the average steady-state means of an endogenous variable \(X\), denoted \(\bar{X}\), is specific to a given industry characterized by a particular pair of parameter values, \((s, f)\). The comparative dynamics analysis entails comparing the values of \(\bar{X}\) for different configurations of \(s\) and \(f\), each representing a specific type of industry. In particular, I consider \(s \in \{3, 4, 5, 6\}\) and \(f \in \{200, 300, 400, 500\}\).

### 5.3.1. Steady-State Volatility of an Industry

The external shocks to the technological environment induce persistent turnover of firms, generating endogenous volatility in the structure of the industry. Such structural volatility has two distinct components: 1) the fluctuation in the size and composition of the industry due to the movements of firms \textit{in and out of the industry}; 2) the fluctuation in the distribution of market shares among firms \textit{within the industry}. The first component characterizes the turbulence at the boundary of an industry. For expositional convenience, I will refer to it as \textit{inter-industry volatility}. In the context of our model, the term, “inter-industry,” reflects the fact that a firm exiting an industry may be viewed as moving to another industry that offers the next best alternative opportunity. Similarly, a new entrant could be viewed as coming from another industry. The second component characterizes turbulence strictly within the industry, in that the focus is on the frequency with which the market dominance changes hands. It reflects the changing fortunes of the incumbent firms caused by the unexpected changes in the technological environment and the stochastic nature of their R&D activities. I will refer to this type of volatility as \textit{intra-industry volatility}.

In my model, the inter-industry volatility is captured by the usual rates of entry and exit (turnover). The intra-industry volatility, however, is more difficult to gauge, as it involves the changing positions of all firms within the industry. In this study, I will focus on a particular aspect of that volatility; namely, the (in-)ability of the market leader – i.e., the firm with the largest market share – to retain its dominant position. There are two equivalent ways to capture this: 1) the number of consecutive periods during which a market leader stays in the leadership position (duration of the leadership); 2) the frequency with which the identity of the leader changes. The latter measure is immediate, once we have a complete record of the leadership durations over the horizon. While both measures provide the same information, the “frequency of leadership changes” will be mainly used in this section to represent the intra-industry volatility.

Figure 19 plots the mean steady-state rates of entry and exit, \(\bar{ER}\) and \(\bar{XR}\), for \(s \in \{3, 4, 5, 6\}\) and \(f \in \{200, 300, 400, 500\}\). As displayed in the figure, both rates decrease with the size of the market \((s)\) and increase with the size of the fixed cost \((f)\). Hence, given a group of heterogeneous industries,
differentiated in terms of the market size and fixed cost, an industry with a high (low) rate of entry is also likely to have a high (low) rate of exit; some industries exhibit inherently greater inter-industry volatility than others. Specifically, an industry serving a smaller market and/or having a higher fixed cost displays a higher degree of inter-industry volatility.

Property 3: The rate of turnover (inter-industry volatility) decreases with the size of the market and increases with the size of the fixed cost.

Since the rate of entry and the rate of exit respond to the parameters in the same way (i.e., both higher or both lower), this result is fully consistent with the finding of DRS (1988) as captured in Figure 18: an industry with a higher (lower) rate of entry also has a higher (lower) rate of exit.

To explore the intra-industry volatility, I track the duration of the market dominance for each leader over the 2,000 periods of the steady state between $t = 3,001$ and $t = 5,000$ for each replication; only the fully completed leadership episodes (those that started after $t = 3,001$ and ended before $t = 5,000$) are counted. This exercise also gives us the number of times that the market leadership changed hands over the steady state for each replication. Taking the mean over the 500 replications, Figure 20 shows that the mean number of leadership changes is higher in an industry with a larger market size ($s$) and/or a smaller fixed cost ($f$).

Property 4: The frequency of leadership changes (intra-industry volatility) increases with the size of the market and decreases with the size of the fixed cost.

Equivalently, the mean duration of the leadership declines with market size ($s$) and increases with fixed cost ($f$).

Let us now consider the relationship between the inter-industry volatility and the intra-industry volatility, based on Figures 19 and 20. Notice from Property 3 and Property 4 that the inter-industry volatility (rate of turnover) and the intra-industry volatility (frequency of leadership changes) are negatively related to each other: An industry with a higher (lower) inter-industry volatility has a lower (higher) intra-industry volatility. For instance, consider two heterogeneous industries; industry A with $s = 6$ and $f = 200$ and industry B with $s = 3$ and $f = 500$. Industry A exhibits the rate of entry at about 1.25% but the market leadership changed about 298 times. In industry B, the rate of entry is around 2.15% and the market leadership changes about 281 times. Hence, industry A shows greater intra-industry volatility, but industry B shows greater inter-industry volatility.

The divergent manner in which the two types of volatility respond to the two parameters indicates that it is inadequate for understanding the underlying dynamics to describe the structural volatility of an industry using only the rate of turnover. An important question is: What drives the difference in the two volatility measures? It turns out that the answer to this question lies in the way R&D influences the selective force of the market competition.

To test the effect of R&D on the industry volatility, let us run the baseline replication under the alternative condition that the firms perform no R&D. Firms are assumed to come into the industry with a randomly chosen technology which then stays fixed for the rest of the firm’s life. The entry and exit of the firms are purely driven by the selective force of the market competition, with no adaptive efforts (i.e., R&D) by the firms. We will compare the outcome from this alternative case to that from the original baseline with endogenous R&D.

Figure 21 reports on the results of this experiment by plotting histograms of the mean steady-state values of the four endogenous variables from the 500 independent replications of the baseline case.
with and without endogenous R&D: a) the rate of entry; b) the rate of exit; c) the duration of industry leadership; and d) the frequency of leadership changes. First, note that the presence of endogenous R&D shifts the rates of entry and exit downward. Hence, there is less inter-industry volatility in the presence of endogenous R&D. Second, endogenous R&D shifts the leadership duration downward and the frequency of leadership changes upward: there is greater intra-industry volatility in the presence of endogenous R&D.

Property 5: Endogenous R&D reduces inter-industry volatility, but increases intra-industry volatility.

Clearly, the pursuit of R&D by firms allows the incumbents to adapt to the changing environment to a greater extent, compared to the situation where they have no such adaptation mechanism. The resulting cost advantage protects the incumbents from the potential entrants who come in with randomly chosen technologies; this leads to the reduced inter-industry volatility under endogenous R&D. However, the improved efficiency of the incumbents, attained through R&D, also increases the degree of competition among themselves as they are better adapted to the new environment on average and have lower and tighter distribution of marginal costs. Such convergence in marginal costs makes it easier for a laggard to leapfrog the leader, reducing the overall duration of the industry leadership (or increasing the frequency of leadership changes) and thus raising the intra-industry volatility.

Further evidence on the effect of R&D on the structural volatility is obtained by examining the intensity of firms’ R&D in the steady state as observed for different combinations of the two parameters, (s and f). There are two measures we look at in this regard. At the industry level, the R&D intensity is captured by the aggregate R&D spending (\( \hat{TRD} \)). Given the two modes of R&D in the model, innovation and imitation, we also look at \( \hat{NRD} \), the share of the innovation cost in the aggregate R&D spending. Figure 22 shows the impact of s and f on the two R&D variables.

Property 6: Both the aggregate R&D spending and the innovation’s share of the R&D expenditure increase with the size of the market and decrease with the size of the fixed cost.

Given the role of endogenous R&D described in Property 5, one would expect a higher aggregate R&D spending (in industries with larger market size and smaller fixed cost) to lead to a lower inter-industry volatility and higher intra-industry volatility and vice versa. The results shown in Figure 22 are fully consistent with this intuition, when viewed in conjunction with Properties 3 and 4.

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19 The cost advantage can emerge for the incumbents even when there is no endogenous R&D, since the selection force of the market competition tends to weed out the relatively inefficient firms from the industry, thereby improving the efficiency of the incumbents on average. The assumption of endogenous R&D further enlarges this relative cost advantage, hence raising the entry barrier, by allowing the incumbents to individually lower their costs as well. This endogenous entry barrier is unlikely to exist when the potential entrants can copy the best available technology of the incumbents at zero cost. But to the extent that the imitation of a technology remains imperfect, the R&D efforts by the incumbents will create a barrier against the potential entrants.

20 It should be noted that the aggregate R&D spending in Property 6 is driven by the endogenous number of firms. As will be shown in the next section, the total number of firms that an industry can sustain increases with the market size and decreases with the size of the fixed cost. When the aggregate R&D spending is divided by the number of firms, the resulting R&D spending per firm responds to the changes in the two parameters in exactly the opposite way: it decreases with the market size and increases with the size of the fixed cost. Since our focus in this section is mainly on the industry-level variables, we stay with the aggregate measure rather than the per-firm measure.
It is also worthwhile to note that the behavior of the share of the innovation cost in the aggregate R&D spending, as shown in Figure 22(b), implies that those industries that exhibit lower inter-industry volatility but higher intra-industry volatility (i.e., those with higher $s$ and lower $f$) are likely to spend a greater share of R&D expenditure on innovation (rather than imitation) than another industry with higher inter-industry volatility but lower intra-industry volatility (i.e., those with lower $s$ and higher $f$).

In summary, the negative relationship between the inter-industry volatility and the intra-industry volatility as implied by Property 3 and Property 4 can be explained on the basis of the differential ways in which endogenous R&D affect these two measures of structural volatility.

5.3.2. Steady-State Structure and Performance of an Industry

The turnovers described in the previous section have a long-run impact on the steady-state number of firms that an industry can sustain. Figure 23 shows that the number of firms (both active and inactive) increases with $s$ and decreases with $f$, while the industry concentration, measured by the Herfindahl-Hirschmann Index, decreases with $s$ and increases with $f$. Hence, those industries with a smaller market size and a larger fixed cost are likely to be more concentrated and have a higher rate of turnover.

Property 7: The industry concentration decreases with the size of the market and increases with the size of the fixed cost.

It is notable that this result is in line with the predictions of the static equilibrium model of oligopoly, in which the free-entry equilibrium number of firms was directly related to the market size and inversely related to the fixed cost. [See equation (10) in Section 2.1.2.]

Figure 24 captures the impacts $s$ and $f$ have on $\bar{P}$ (market price), $\bar{WMC}$ (industry marginal cost), and $\bar{PCM}$ (industry price-cost margin). All three variables are negatively related to the size of the market and positively related to the fixed cost.

Notice that the market price and the industry price-cost margin are related to the market size and the fixed cost in the same way that the industry concentration is related to the two parameters: They decrease with the market size and increase with the fixed cost. This implies that the price and the price-cost margin are both positively related to market concentration. While this is consistent with the market power explanation behind the concentration-price relationship (or the concentration-price-cost margin relationship), Figure 23(a)-(b) and Figure 24(b) jointly suggest an alternative explanation. Those industries with smaller market size and larger fixed cost are capable of sustaining a smaller number of firms. The lower degree of competition in a relatively concentrated industry implies a weaker selection pressure on the incumbent firms. Consequently, the steady-state level of the industry marginal cost tends to be higher due to the relative inefficiency of the firms in smaller markets (or the markets with higher fixed cost). This leads to a higher market price. As shown in Figure 24(c), the price-cost margin is also higher in those industries with smaller market size and higher fixed cost. These results then suggest that the market power effect and the efficiency effect jointly determine the relationship between industry concentration and the market price (or the price-cost margin).

Property 8: Both the market price and the industry marginal cost decrease with the size of the market and increase with the size of the fixed cost. The industry price-cost margin also decreases with the size of the market and increases with the size of the fixed cost.
It is straightforward to establish the relationships between the endogenous variables on the basis of the properties established above. For instance, an industry with a high rate of firm turnover is likely to be highly concentrated. The market price in such an industry is also likely to be high. However, the firms are likely to invest less heavily in R&D, with a relatively greater emphasis on imitation than innovation. The same industry tends to have higher industry marginal cost (and thus be relatively inefficient), but generate higher price-cost margins for the firms. These predicted relationships have implications for cross-sectional empirical research as discussed next.

5.4. Implications for Cross-Industries Studies

Traditional empirical studies in industrial organization, addressing the issues of market structure and performance, are cross-sectional studies of a large number of heterogeneous industries. [See Schmalensee (1989) and Caves (2007) for surveys of this immense literature.] These industries are likely to vary widely in terms of the size of the market they operate in as well as the size of the fixed cost that determines the economies of scale for the firms within each industry. Given the variety of industries, the past empirical studies attempted to identify relationships between variables that are endogenous to the industry dynamics, such as the rate of firm turnover, industry concentration, market price, price-cost margins, and R&D intensities. The comparative dynamics results presented in the previous section can assist us in understanding these cross-sectional relationships within a unifying conceptual framework.

In the context of our model, industries can differ in terms of the two parameters, \( s \) and \( f \). Table 7 summarizes the relationships between the two parameters and the major endogenous variables as identified in our comparative dynamics exercise in Section 5.3.

There are seven endogenous variables that have the same relationship with respect to the size of the market and the fixed cost: the rate of entry, the rate of exit, industry concentration, duration of industry leadership, market price, industry marginal cost, and industry price-cost margin are all negatively related to \( s \) and positively related to \( f \). Conversely, the remaining four variables — the number of firms, the frequency of leadership changes, the aggregate R&D spending, and the cost share of innovation in aggregate R&D — are positively related to \( s \) and negatively related to \( f \).

One may infer from Table 7 the implicit relationships between the endogenous variables that have been central to many of the cross-sectional studies in industrial organization. First, the number of firms and market price are negatively related (or alternatively the industry concentration and market price are positively related). Similarly, the number of firms and the industry price-cost margin are negatively related (i.e., the industry concentration and the industry price-cost margin are positively related).

Property 9: Industry concentration is positively related to market price and the industry price-cost margin.

Both of these predictions are consistent with the traditional structure-conduct-performance paradigm that drove much of the cross-sectional studies in the empirical literature. [Weiss (1989) provides a collection of empirical studies that examine these issues in detail. Also see Schmalensee (1989) for a comprehensive survey of the empirical literature in this tradition.]

Second, the rate of entry and the rate of exit are positively related so that an industry with a higher than average rate of entry also has a higher than average rate of exit — i.e., there are high turnover
industries and low turnover industries. [DRS (1988)] Furthermore, the rate of turnover is positively related to industry concentration so that a concentrated industry tends to have high rate of firm movements in and out of the industry.\(^\text{21}\)

*Property 10:* The rate of turnover is positively related to industry concentration.

This also implies that the duration of industry leadership is positively related to industry concentration, while the frequency of leadership changes is negatively related to industry concentration. Hence, market dominance by a single firm is likely to be longer-lasting in a more concentrated industry.

*Property 11:* The duration of industry leadership is positively related to industry concentration.

Third, a high turnover industry tends to have relatively inefficient firms, but the industry price-cost margin tends to be high due to the high market price.

*Property 12:* The rate of turnover is positively related to industry price-cost margin.

Both the R&D intensity as measured by the aggregate R&D spending and the cost share of innovative R&D (rather than imitative R&D) tend to be low in those industries having high turnovers (high inter-industry volatility). Conversely, both R&D variables are high in those industries where the leadership changes are frequent (i.e., high intra-industry volatility).

*Property 13:* The aggregate R&D spending and the cost share of innovative R&D are negatively related to the rate of turnover, but positively related to the frequency of leadership changes.

### 6. FIRM DYNAMICS IN THE STEADY STATE: WITHIN-INDUSTRY VARIATIONS

Section 5 was devoted to studying the aggregate (or average) behavior of firms over time and across industries through the comparative dynamics analysis of the steady states. In this section, I go beyond the aggregate behavior and probe deeper into the behavior of individual firms in order to identify any persistent heterogeneity that exist among firms in the same industry. I start by focusing on two types of heterogeneities: 1) technological diversity among firms; and 2) the inequality in market shares held by the competing firms. Variations along these two dimensions are driven by the persistent entries and exits of firms over time as well as the evolving asymmetry in their production efficiencies. This also implies that the firms vary widely in terms of their survivability. The last part of the section addresses the variation in firms’ life span and the infant mortality phenomenon it implies.

#### 6.1. Technological Diversity

**6.1.1. R&D Reduces Technological Diversity**

To motivate the research questions posed in this section, I start with the observation made in Section 3 that a typical (baseline) industry in this model displays a persistent degree of technological diversity.

\(^{21}\) It should be noted that this result is specific to the rate of entry or exit. The level (number) of entry or exit is smaller in a more concentrated industry.
diversity as defined in equation (23) – see Figure 6(a)-(b). Such persistence in technological diversity is driven by the external shocks to the technological environment. Note that a firm’s pursuit of R&D (whether it is innovative or imitative) is adaptive in nature since the R&D activity is modeled as a search process in which all firms try to move closer to the unique optimal technology (common to all firms). Given the latest shock to the technological environment, the gradual adaptation (via R&D) by firms reduces the degree of technological diversity within the industry until it is hit by another external shock. When a new technological shock hits, it offers new opportunities for the potential entrants to come in with technologies different from those of the incumbents, temporarily raising the degree of technological diversity in the industry. The mean degree of technological diversity is then the result of the balancing act between the two countervailing forces: 1) the external shocks to the technological environment that raise the degree of diversity through entry of new firms; and 2) the R&D activities of the incumbents that reduce the degree of diversity through improved adaptation to the new technological environment.

In view of the above forces, making firms’ R&D endogenous in the model, while holding the rate ($\gamma$) and the magnitude ($\varphi$) of the external shocks fixed, should decrease the mean degree of technological diversity ($DIV$) along the steady state. I perform a computational experiment based on our model to confirm this intuition. First, for the baseline configuration of the industry parameters, I generate and grow an industry with the usual specification that all firms make their R&D decisions endogenously. For the same parameter configuration, I then generate and grow an alternative industry in which firms enter, each endowed with a random technology, but then never perform any R&D (i.e., never adopt any new technology) during their stay in the industry: Hence, the component of the model that allows for endogenous R&D is turned off for this alternative industry.

Under the two specifications I compute the steady-state degree of technological diversity (mean of the diversity measure over the last 2,000 periods between $t = 3,001$ and $t = 5,000$) for each replication. Figure 25 shows the histograms of the steady-state means with and without endogenous R&D from 500 independent replications. In line with the intuition provided above, allowing firms to perform R&D reduces the mean degree of technological diversity.

Given the role of endogenous R&D as described above, one could further conjecture that, when R&D is endogenous, the reduction in the cost of R&D ($\varphi_{IN}$ and $\varphi_{IM}$) will lead to a decline in the steady state diversity ($DIV$) by inducing firms to raise the aggregate volume of R&D activities. This is verified in a comparative dynamics exercise performed with respect to the fixed costs of R&D (i.e., $K_{IN}$ and $K_{IM}$), holding all other parameters at their baseline levels.

Figure 26 presents the histograms of the steady state means of the volume of R&D activities as well as those of technological diversity for all 500 replications for four different pairs of $K_{IN}$ and $K_{IM}$, where $K_{IN}$ is always twice as large as $K_{IM}$: $(K_{IN}, K_{IM}) \in \{(100, 50), (300, 150), (500, 250), (700, 350)\}$. First, note that the total R&D spending ($TRD$) is no longer an adequate measure of R&D efforts in this analysis, as it involves varying the costs of R&D themselves: the change in $TRD$ may be due to the change in the volume of R&D activities or that in the costs of R&D. To avoid this problem, we look at the total number of R&D directly (i.e., we count the total number of firms that incurred R&D expense, whether innovative or imitative R&D, at each point in time). Figure 26(a) presents the histograms of the steady state mean number of R&D for the four different cases of $(K_{IN}, K_{IM})$. As expected, the firms are more active in R&D when the costs of R&D are lower. Also in line with our intuition, Figure 26(b) shows that the distribution of steady state mean diversity ($DIV$) shifts up as the costs of R&D rise. Based on these observations, we conclude:
Property 14: Lower costs of R&D induce higher volume of R&D activities; consequently, the more intensive search results in reduced technological diversity along the steady state.

6.1.2. Impact of Market Size and Fixed Cost on Technological Diversity

The degree of technological diversity also depends on the values of the two main parameters of the model, $s$ and $f$. Figure 27 shows that the average value of the steady-state means, $\overline{DIV}$, over 500 replications stays above 0.448 for all parameter values. Furthermore, the following property is identified:

Property 15: The mean technological diversity ($\overline{DIV}$) increases with the size of the market ($s$) and decreases with the size of the fixed cost ($f$).

Hence, the firms are likely to hold more diverse technologies in larger markets and in those markets with relatively lower fixed cost of production. Since the steady-state number of firms is higher in these markets, the higher degree of technological diversity is driven mainly by the larger population size.

6.2. Market Share Inequality

The diversity among technologies adopted by firms within a given industry leads to asymmetries in the production efficiencies of these firms. This also implies that there must be significant variation in the market shares held by these firms. The extent of such inequality in any given period may be measured using the Gini coefficient ($G^s$) as defined in equation (24) in Section 3.1. In this section, we examine how endogenous R&D as well as the industry-specific factors such as $s$ and $f$ may affect the steady state mean inequality, $\overline{G}$.

6.2.1. R&D Increases Market Share Inequality

Unequal market shares indicate differences in production efficiencies of the competing firms. Since the long-run efficiency of a firm depends on the extent to which it carries out its R&D activities, a relevant question is how allowing firms to perform R&D affects the steady-state degree of market share inequality. To find an answer to this question, we repeat the computational experiment of the type performed in the previous section, in which two sets of simulations are run, one without endogenous R&D and one with endogenous R&D. Each set of simulations contains 500 independent replications. The steady state mean of the Gini coefficient, $\overline{G}$, is computed for each of these replications. The histograms of $\overline{G}$ for these 500 replications for the two cases are presented in Figure 28. As clearly shown, the market shares are more unequally distributed (i.e., Gini coefficient is higher) when firms are allowed to perform R&D.

Recall from the discussion of “inter-industry volatility” in Section 5 that the pursuit of R&D allows the incumbent firms to adapt to the changing environment more effectively (relative to potential entrants); hence the reduced inter-industry volatility under endogenous R&D. The general improvement in efficiency of the incumbents, however, increases the degree of competition among themselves, resulting in a lower and tighter distribution of marginal costs for the surviving firms: A slight cost advantage is likely to give the firm a substantial increase in its market share, thus increasing the market share inequality along the steady state.

The above intuition is further supported by the comparative dynamics exercise performed with respect to $K_{IN}$ and $K_{IM}$. Repeating the procedure from the previous section, we compute the steady
state mean of the Gini coefficient for 500 independent replications for $(K_{IN}, K_{IM}) \in \{(100,50), (300, 150), (500, 250), (700, 350)\}$. The results are shown in Figure 29. In line with our intuition, we observe the following property:

**Property 16:** The market share inequality is higher, when the cost of R&D is lower (and, hence, the volume of R&D is greater).

### 6.2.2. Impact of Market Size and Fixed Cost on Market Share Inequality

Market share inequality also depends on the size of the market $(s)$ and the fixed cost $(f)$. The comparative dynamics results with respect to the two parameters are shown in Figure 30.

**Property 17:** The market share inequality $(\bar{G})$ increases with the size of the market $(s)$ and decreases with the size of the fixed cost $(f)$.

Properties 15 and 17 jointly indicate that technological diversity $(DIV)$ and market share inequality $(\bar{G})$ are positively related when industries are differentiated on the basis of the market size and the fixed cost: those values of $s$ and $f$ that lead to high technological diversity also lead to high market share inequality.

### 6.3. Life Span of Firms

The persistent entry and exit that characterize the steady state of the industry have an implication for the life span of firms. An industry that has a high rate of turnover should display relatively shorter life span for an average firm and vice versa. Before engaging in a comparative study of the between-industry variations in this regard, let us first examine the variation in the life span of firms operating within a given industry.

As a point of departure, I suggest that we go back to the entry/exit data from the US automobile industry between 1895 and 1966 (from Section 4). The entry and exit data, constructed from Smith (1968), were plotted in Figure 8. There were 917 total exits over the period, but there were also wide variation in the ages of the firms at the time of their exit. Some firms exited the same year they entered the market, in which case their life span is zero; others were more fortunate, although, as shown in Figure 9, the phenomenon of infant mortality is clearly in display during this time period. The maximum life span was observed to be 62 years.

Given the data on the ages of firms at the time of their exit, let us produce a histogram of the exit ages by binning them into bins of equal size 5. That is, the first bin goes from 0 to 4, the second from 5 to 9, the third from 10 to 14, and so forth. The resulting histogram is shown in Figure 31 (a), where the horizontal axis shows the starting age for each consecutive bin and the vertical axis captures the fraction of all exiting firms which were of ages belonging to each bin. The figure shows that 63% of all exiting firms had the life span between 0 and 4, 24% had the life span between 5 and 9, 6% had the life span between 10 and 14, and so forth.

Figure 31(b) plots the same histogram on log-log scale. The plot on logarithmic scale is approximately linear, which reveals the power-law form of the distribution underlying the life span of firms. To be more specific, let $f(x)dx$ be the fraction of exiting firms with ages between $x$ and $x + dx$. The fact that the histogram is a straight line on log-log scale implies that $\ln f(x) = -\hat{a} \ln x + \hat{c}$, where $\hat{a}$ and $\hat{c}$ are constants. Taking the exponential of both sides, this can be rewritten as:
\[ f(x) = \hat{C} x^{-\hat{a}}, \quad (7.1) \]

where \( \hat{C} = e^\hat{c} \). Distributions that have the above form are said to follow power law, where the constant \( \hat{a} \) is referred to as the exponent of the power law.

Given the age-at-exit data from the auto industry, a least-squares fit can be found, where the best fit entails \( \hat{a} = 2.83755 \) and \( \hat{c} = 0.12509 \). The fitted line is super-imposed on the actual data in Figure 32. The power law distribution appears to characterize the life span of the firms in the US auto industry quite well.

The next question is whether our computational model is capable of predicting this property. To pursue this question, let us randomly pick a replication from the baseline simulations and examine the ages of the exiting firms at the time of each exit that takes place over the steady state between \( t = 3,001 \) and \( t = 5,000 \). Figure 33 (a) shows a histogram of the ages at exit (i.e., life span) for a randomly chosen replication; it had a total of 1,363 exits over the 2,000 periods of steady state. The binning of the ages here is at its most refined such that each bin is of size one. There were 174 instances (13%) where exiting firms were exactly 1 year old; 176 instances (13%) where they were 2 years old; 85 instances (6%) where they were 3 years old; 55 instances (4%) where they were 4 years old; and so forth. The phenomenon of “infant mortality” is clearly visible in this figure.

More interestingly, when the same histogram is plotted on log-log scale in Figure 33(b), it takes the general shape of a straight line with a negative slope as in the case of the US auto industry. The data toward the tail-end of the distribution are quite noisy, however, and this is due to the bins being of such a small size. To get a smoother plot, we can increase the size of the bin. For instance, suppose we bin the data such that each bin contains exactly 200 age levels: the first bin contains ages, 0 to 199, the second bin contains ages, 200 to 399, and so forth. The resulting histogram is shown in Figure 34(a) as a linear plot and 34(b) as the log-log plot.

Just as in the case of the US automobile industry, the model predicts the life span of firms to be characterized by power law distribution. The best fit for the data captured in Figure 34 entails \( \hat{a} = 3.87739 \) and \( \hat{c} = 0.003927 \). The fitted line is plotted with the model-generated data in Figure 35. The fit is almost perfect.

In line with the previous analyses on diversity and inequality, I now consider the impact endogenous R&D has on the distribution of firms’ ages at the time of exit. I collect the age-at-exit data from the 500 independent replications performed under two separate specifications, one where R&D is made endogenous and one where R&D is completely turned off.

The collected data from a random replication under the two specifications are plotted together on log-log scale in Figure 36. Notice that it indicates the distribution of age-at-exit (life span) as being lower when firms do not perform any R&D. A careful examination of the data for all 500 replications under each specification confirms that this is a general property. In fact, the mean exponent of the power law (i.e., averaged over 500 replications) is \( \hat{a} = 3.76913 \) (0.295897) with R&D and is \( \hat{a} = 5.23248 \) (0.349334) without R&D. [The number inside the parenthesis is the standard deviation.] Hence, endogenous R&D increases the life span of a firm.

We can also see the impact of the two parameters, \( f \) and \( s \), on the exponent of the power law (\( \hat{a} \)). Table 8 reports the exponent, averaged over 500 replications, for each parameter configuration.
(with the standard deviation inside the parentheses). We observe that the exponent decreases with the size of the market \((s)\) and increases with the size of the fixed cost \((f)\).

To get a more intuitive perspective on how the two parameters, \(f\) and \(s\), affect the life span of firms, I further derive the cumulative density of a given age-at-exit by asking what proportion of all exiting firms exited at a given age (AGE) or younger – this is the same type of information that was displayed in Figure 9 for the US auto industry. This information is plotted in Figure 37(a) for \(f \in \{200,300,400,500\}\) given \(s = 4\); and Figure 37(b) for \(s \in \{3,4,5,6\}\) given \(f = 200\). Consistent with the observation on the power law exponent, the life span of a firm tends to be shorter (i.e., a larger proportion of firms exit at a given age or younger) when the fixed cost is higher and/or the market size is smaller.

Property 18: The average life span of a firm decreases with the size of the market \((s)\) and increases with the size of the fixed cost \((f)\).

Recall from Property 3 that the rate of firm turnover decreases with the size of the market and increases with the size of the fixed cost. Property 18 follows from that observation in that the average life span of a firm should be shorter in those industries with higher rates of firm turnover.

7. CONCLUSION

The Schumpeterian process of creative destruction is often conceptualized as the Darwinian evolutionary process. The model of industry dynamics proposed here can be viewed in a similar framework. It has two interacting mechanisms that jointly determine how a given industry will evolve over time. The first is the mechanism that generates and maintains technological variation in the population of firms (and, hence, the varying degrees of production efficiency on the supply-side). What guarantees the generation and maintenance of such variation is the inherent tendency of the firms to pursue available profit opportunities, not only in their choices of output, but in the persistent entry by new firms and the R&D efforts of the existing firms. Additionally, the exogenous shocks to the technological environment provide a continuous incentive for firm R&D even in the long run. The second mechanism, interacting with the first, is the market competition of sufficient severity that induces the survival of only a subset of firms from the existing population, where the survival advantage goes to those firms with the technologies giving the lowest cost in the current environment. It is the continuing interaction of these two mechanisms that drive the process of industrial dynamics in the model presented here.

The evolutionary process of Schumpeterian competition was implemented in this study using an agent-based computational model, in which artificial industries were created and grown to maturity in silico. While the firms in the model were assumed to have bounded rationality, they were nevertheless adaptive in the sense that their experience-based R&D efforts allowed them to search for improved technologies. In the early phase the means and variances of industry data change, but the external shocks that perennially occur never allow the industry to achieve a truly “static” equilibrium. Given a technological environment subject to persistent and unexpected external shocks, the computationally-generated industry remains in a perennial state of flux. More precisely, once past the transient growth phase, it reaches a steady state in which the measured behavior of the firms and the industry stochastically fluctuate around steady means. The main objective of the study was to identify the patterns that exist in the movements of the firms as the industry evolves over time along the steady state. The detailed investigation of the non-equilibrium adjustment dynamics by firms, as carried out here, is not feasible with the standard equilibrium IO models.
The ultimate value of any model, of course, depends on its capacity to make predictions that can match data. I started the paper with four stylized facts from the empirical IO literature: 1) shakeouts in infant industries; 2) persistent entries and exits in mature industries; 3) positive correlation between the entry and the exit time series of a typical industry; and 4) positive correlation between entry and exit across different industries. The computational model developed in this study was able to replicate all of these phenomena, both along the transient growth phase and the steady state phase of the industry. Furthermore, I was able to identify other patterns inherent in the steady-state dynamics of a given industry, as well as variations in those patterns across industries with different characteristics, by systematically varying the characteristics of the market and the technological environment within which the computationally-generated industry evolves. This was implemented by performing comparative dynamics analyses on various endogenous variables with respect to the two main parameters of the model, the size of the market \(s\) and the fixed cost \(f\). The following results were generated from these analyses, many of which are consistent with the existing empirical literature.

First, in the realm of results that could fall under the rubric of comparative statics, those industries with a higher market size and/or lower fixed cost, call them HSLF industries, were shown to have a larger carrying capacity such that the endogenous number of firms in the steady state is larger; hence these industries are less concentrated than those with a smaller market size and/or a higher fixed cost. Secondly, the HSLF industries exhibit lower market price and lower price-cost margins along the steady state, thus predicting a positive relationship between concentration and price (or price-cost margins).

More significantly, our model produced results on the dynamics of firms and the industry. The HSLF industries are externally less turbulent in that they have a lower rate of turnover. However, these industries are also internally more turbulent in that they have more frequent changes in the market leadership. The distinct ways in which the two industry-specific factors affect the inter- and intra-industry volatilities can be explained through the R&D activities of firms. The firms in HSLF industries spend more on R&D on aggregate (and also invest more on innovative R&D than imitative R&D) along the steady state. The more intensive R&D efforts endogenously create an entry barrier by improving the average efficiency of the incumbents relative to that of the potential entrants. This endogenous entry barrier reduces the inter-industry volatility, protecting the incumbents from potential competition. The other side of the same coin, however, is that the more intensive R&D, which is adaptive in nature, also leads to more intense competition among incumbents by reducing their technological diversity. The consequent narrowing of the efficiency differentials results in higher intra-industry volatility.

Given the persistent technological diversity and the perpetual entry and exit of firms over time, there are substantial variations in the efficiency of firms within an industry at any given point in time. These variations, in turn, have implications for the survivability of technologically heterogeneous firms. In third set of results, consistent with the empirical observations, our model predicts a wide variation in the life span of firms, distribution of which can be characterized by the power law distribution. Furthermore, a close look at the ages of the exiting firms indicates a clear case of infant mortality phenomenon, and the comparative dynamics analysis predicts the average life span of a firm to be longer in those industries with a larger market size and/or a lower fixed cost (HSLF). The latter result is consistent with the earlier result that the inter-industry volatility is lower in the HSLF industries.

The computational model of industry dynamics developed here lends itself to further extensions. There are two important aspects of industry dynamics that are missing in the current version of the model but would be important to address. The first is the option for firms to merge with one another;
the second is the role of patents in the Schumpeterian process of “creative destruction.” These features are undoubtedly central to the current normative debates on the antitrust and technology policies. Although I have not taken the step into this territory in this paper, I hope to have provided an appropriate foundation for further computational experiments that could allow normative policy analysis with respect to these important features.

REFERENCES


### Table 1: Set Notations

<table>
<thead>
<tr>
<th>Notation</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S^t$</td>
<td>Set of surviving firms at the end of $t$</td>
</tr>
<tr>
<td>$S^t_π$</td>
<td>Those in $S^t$ which were profitable in $t$</td>
</tr>
<tr>
<td>$R^t$</td>
<td>Set of potential entrants at the beginning of $t$</td>
</tr>
<tr>
<td>$E^t$</td>
<td>Set of actual entrants in $t$</td>
</tr>
<tr>
<td>$M^t$</td>
<td>Set of firms poised to compete in $t$ $(= S^{t-1} \cup E^t)$</td>
</tr>
<tr>
<td>$L^t$</td>
<td>Set of firms which exit the industry at the end of $t$</td>
</tr>
</tbody>
</table>

### Table 2: Beliefs underlying the Firms’ Decision-making

<table>
<thead>
<tr>
<th>Potential Entrant ($k$)</th>
<th>All $k \in R^t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Given:</td>
<td>Market size: $s$</td>
</tr>
<tr>
<td></td>
<td>Technological environment: $\mathcal{E}^t$</td>
</tr>
<tr>
<td></td>
<td>Endowed technology: $\mathcal{E}^t_k$</td>
</tr>
<tr>
<td>Beliefs:</td>
<td>Set of players: $S^{t-1} \cup k$</td>
</tr>
<tr>
<td></td>
<td>Marginal costs: $c^t_i \forall i \in S^{t-1}$ and $c^t_k$ for $k$</td>
</tr>
<tr>
<td>Compute:</td>
<td>Expected profit: $\pi^t_k(\mathcal{E}^t_k)$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Surviving Incumbents ($j$)</th>
<th>All $j \in S^{t-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Given:</td>
<td>Market size: $s$</td>
</tr>
<tr>
<td></td>
<td>Technological environment: $\mathcal{E}^t$</td>
</tr>
<tr>
<td></td>
<td>Current technology: $\mathcal{E}^{t-1}_j$</td>
</tr>
<tr>
<td>Beliefs:</td>
<td>Set of players: $S^{t-1}$</td>
</tr>
<tr>
<td></td>
<td>Marginal costs: $c^t_{j-1} \forall i \in S^{t-1}$</td>
</tr>
<tr>
<td>Compare:</td>
<td>$c^t_{j-1}$ and $\hat{c}^t_j$</td>
</tr>
</tbody>
</table>

### Table 3: Evolving Attractions

<table>
<thead>
<tr>
<th>Decision Path</th>
<th>Updating of Attractions</th>
</tr>
</thead>
<tbody>
<tr>
<td>No R&amp;D</td>
<td>$A^t_{i+1} = A^t_i$; $\bar{A}^t_{i+1} = \bar{A}^t_i$; $B^t_{i+1} = B^t_i$; $\bar{B}^t_{i+1} = \bar{B}^t_i$;</td>
</tr>
<tr>
<td>R&amp;D Innvate</td>
<td>$A^t_{i+1} = A^t_i + 1$; $\bar{A}^t_{i+1} = \bar{A}^t_i$; $B^t_{i+1} = B^t_i + 1$; $\bar{B}^t_{i+1} = \bar{B}^t_i$;</td>
</tr>
<tr>
<td>Discard</td>
<td>$A^t_{i+1} = A^t_i$; $\bar{A}^t_{i+1} = \bar{A}^t_i + 1$; $B^t_{i+1} = B^t_i$; $\bar{B}^t_{i+1} = \bar{B}^t_i + 1$;</td>
</tr>
<tr>
<td>R&amp;D Imitate</td>
<td>$A^t_{i+1} = A^t_i + 1$; $\bar{A}^t_{i+1} = \bar{A}^t_i + 1$; $B^t_{i+1} = B^t_i + 1$; $\bar{B}^t_{i+1} = \bar{B}^t_i + 1$;</td>
</tr>
<tr>
<td>Discard</td>
<td>$A^t_{i+1} = A^t_i$; $\bar{A}^t_{i+1} = \bar{A}^t_i + 1$; $B^t_{i+1} = B^t_i + 1$; $\bar{B}^t_{i+1} = \bar{B}^t_i$;</td>
</tr>
</tbody>
</table>
Table 4: List of Parameters and Their Values

<table>
<thead>
<tr>
<th>Notation</th>
<th>Definition</th>
<th>Baseline Value</th>
<th>All Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>Number of tasks</td>
<td>96</td>
<td>96</td>
</tr>
<tr>
<td>$r$</td>
<td>Number of potential entrants per period</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>$b$</td>
<td>Start-up wealth for a new entrant</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$W$</td>
<td>Threshold net wealth for survival</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$a$</td>
<td>Demand intercept</td>
<td>300</td>
<td>300</td>
</tr>
<tr>
<td>$f$</td>
<td>Fixed production cost</td>
<td>200</td>
<td>{200, 300, 400, 500}</td>
</tr>
<tr>
<td>$K_{IN}$</td>
<td>Fixed cost of innovation</td>
<td>100</td>
<td>{100, 300, 500, 700}</td>
</tr>
<tr>
<td>$K_{IM}$</td>
<td>Fixed cost of imitation</td>
<td>50</td>
<td>{50, 150, 250, 350}</td>
</tr>
<tr>
<td>$A_i^0$</td>
<td>Initial attraction for R&amp;D</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>$A_i^P$</td>
<td>Initial attraction for No R&amp;D</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>$B_i^0$</td>
<td>Initial attraction for innovation</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>$B_i^0$</td>
<td>Initial attraction for imitation</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>$T$</td>
<td>Time horizon</td>
<td>5,000</td>
<td>5,000</td>
</tr>
<tr>
<td>$s$</td>
<td>Market size when demand does not fluctuate</td>
<td>4</td>
<td>{3, 4, 5, 6}</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Rate of change in technological environment</td>
<td>0.1</td>
<td>{0, 0.1}</td>
</tr>
<tr>
<td>$g$</td>
<td>Maximum magnitude of change in technological environment</td>
<td>8</td>
<td>8</td>
</tr>
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Table 5: Steady-State Means of the Endogenous Variables*

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
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<tbody>
<tr>
<td>$E_t$</td>
<td>0.684</td>
<td>0.0606</td>
</tr>
<tr>
<td>$L_t$</td>
<td>0.683</td>
<td>0.0606</td>
</tr>
<tr>
<td>$M_t$</td>
<td>41.170</td>
<td>0.4837</td>
</tr>
<tr>
<td>$ER_t$</td>
<td>0.016</td>
<td>0.0015</td>
</tr>
<tr>
<td>$XR_t$</td>
<td>0.016</td>
<td>0.0015</td>
</tr>
<tr>
<td>$p_t$</td>
<td>45.924</td>
<td>0.2108</td>
</tr>
<tr>
<td>$Q_t$</td>
<td>1,016.300</td>
<td>0.8433</td>
</tr>
<tr>
<td>$\Pi_t$</td>
<td>-53.840</td>
<td>113.4550</td>
</tr>
<tr>
<td>$PCM_t$</td>
<td>0.198</td>
<td>0.0019</td>
</tr>
<tr>
<td>$WMC_t$</td>
<td>36.842</td>
<td>0.2335</td>
</tr>
<tr>
<td>$H_t$</td>
<td>357.458</td>
<td>2.6171</td>
</tr>
<tr>
<td>$TRD_t$</td>
<td>1,051.320</td>
<td>17.2790</td>
</tr>
<tr>
<td>$NRD_t$</td>
<td>0.644</td>
<td>0.0025</td>
</tr>
<tr>
<td>$CS_t$</td>
<td>129,114.000</td>
<td>214.6020</td>
</tr>
<tr>
<td>$TS_t$</td>
<td>129,060.000</td>
<td>261.6590</td>
</tr>
<tr>
<td>$DIV_t$</td>
<td>0.453</td>
<td>0.0027</td>
</tr>
<tr>
<td>$G_t$</td>
<td>0.379</td>
<td>0.0091</td>
</tr>
</tbody>
</table>

*These are the descriptive statistics of the steady-state means from the 500 independent replications. Each steady-state mean is the average over the 2,000 periods between $t = 3,001$ and $5,000$. 
Table 6: Correlations between Endogenous Variables (Average over 500 independent replications)

|       | $ER^t$ | $XR^t$ | $|M^t|$ | $P^t$ | $Q^t$ | $II^t$ | $PCM^t$ | $WMC^t$ | $H^t$ | $TRD^t$ | $NRD^t$ | $CS^t$ | $TS^t$ | $DIV^t$ | $G^t$ |
|-------|--------|--------|---------|-------|-------|--------|---------|---------|-------|--------|--------|-------|-------|--------|-------|
| $ER^t$ | 1      |        |         |       |       |        |         |         |       |        |        |       |       |        |       |
| $XR^t$ | 0.321  | 1      |         |       |       |        |         |         |       |        |        |       |       |        |       |
| $|M^t|$  | 0.262  | 0.264  | 1       |       |       |        |         |         |       |        |        |       |       |        |       |
| $P^t$  | 0.318  | 0.208  | 0.340   | 1     |       |        |         |         |       |        |        |       |       |        |       |
| $Q^t$  | -0.318 | -0.208 | -0.340  | -1    | 1     |        |         |         |       |        |        |       |       |        |       |
| $II^t$ | -0.276 | -0.281 | -0.928  | -0.337| 0.337 | 1      |         |         |       |        |        |       |       |        |       |
| $PCM^t$| -0.329 | -0.260 | -0.632  | -0.530| 0.530 | 0.831  | 1       |         |       |        |        |       |       |        |       |
| $WMC^t$| 0.368  | 0.260  | 0.518   | 0.923 | -0.923| -0.606 | -0.814  | 1       |       |        |        |       |       |        |       |
| $H^t$  | -0.225 | -0.200 | -0.565  | -0.103| 0.103 | 0.799  | 0.895   | -0.475  | 1     |        |        |       |       |        |       |
| $TRD^t$| 0.050  | 0.172  | 0.490   | 0.173 | -0.173| -0.585 | -0.310  | 0.258   | -0.274| 1      |        |       |       |        |       |
| $NRD^t$| 0.003  | 0.003  | 0.018   | 0.011 | -0.011| -0.076 | -0.016  | 0.015   | -0.012| 0.344  | 1      |       |       |        |       |
| $CS^t$ | -0.318 | -0.208 | -0.341  | -1.000| 1.000 | 0.338  | 0.530   | -0.923  | 0.103 | -0.173| -0.011| 1      |       |       |        |       |
| $TS^t$ | -0.362 | -0.362 | -0.791  | -0.799| 0.799 | 0.834  | 0.839   | -0.926  | 0.569 | -0.474| -0.055| 0.800  | 1      |       |        |       |
| $DIV^t$| 0.246  | 0.253  | 0.692   | 0.430 | -0.430| -0.651 | -0.496  | 0.517   | -0.356| 0.399 | 0.019 | -0.431 | -0.667 | 1      |        |       |
| $G^t$  | 0.140  | 0.162  | 0.787   | 0.345 | -0.345| -0.536 | -0.124  | 0.291   | 0.036 | 0.391 | 0.013 | -0.345 | -0.544 | 0.591  | 1      |
Table 7: Relationships between the Industry-Specific Factors and the Endogenous Variables

<table>
<thead>
<tr>
<th>Endogenous Variables</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate of Entry ($ER$)</td>
<td>$s$</td>
</tr>
<tr>
<td>Rate of Exit ($XR$)</td>
<td>$f$</td>
</tr>
<tr>
<td>No. Firms</td>
<td>$+$</td>
</tr>
<tr>
<td>Industry Concentration ($\bar{H}$)</td>
<td>$-$</td>
</tr>
<tr>
<td>Duration of Industry Leadership</td>
<td>$-$</td>
</tr>
<tr>
<td>Frequency of Leadership Changes</td>
<td>$+$</td>
</tr>
<tr>
<td>Price ($\bar{p}$)</td>
<td>$+$</td>
</tr>
<tr>
<td>Industry Marginal Cost ($WMC$)</td>
<td>$-$</td>
</tr>
<tr>
<td>Industry Price-Cost Margin ($PCM$)</td>
<td>$-$</td>
</tr>
<tr>
<td>Aggregate R&amp;D Spending ($\bar{TRD}$)</td>
<td>$+$</td>
</tr>
<tr>
<td>Share of Innovation in Aggregate R&amp;D ($\bar{NRD}$)</td>
<td>$-$</td>
</tr>
</tbody>
</table>

Table 8: Fitted Value of the Exponent for the Power Law ($\hat{a}$)

\[
\ln f(x) = -\hat{a} \ln x - \hat{c}
\]

<table>
<thead>
<tr>
<th>$s$</th>
<th>$f = 200$</th>
<th>$f = 300$</th>
<th>$f = 400$</th>
<th>$f = 500$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>4.02977</td>
<td>4.29375</td>
<td>4.44048</td>
<td>4.51863</td>
</tr>
<tr>
<td></td>
<td>(0.305441)</td>
<td>(0.353486)</td>
<td>(0.384022)</td>
<td>(0.423841)</td>
</tr>
<tr>
<td>4</td>
<td>3.76913</td>
<td>4.04658</td>
<td>4.21939</td>
<td>4.34876</td>
</tr>
<tr>
<td></td>
<td>(0.295897)</td>
<td>(0.323955)</td>
<td>(0.362293)</td>
<td>(0.376611)</td>
</tr>
<tr>
<td>5</td>
<td>3.56886</td>
<td>3.85695</td>
<td>4.09201</td>
<td>4.20512</td>
</tr>
<tr>
<td></td>
<td>(0.234936)</td>
<td>(0.304432)</td>
<td>(0.322524)</td>
<td>(0.361473)</td>
</tr>
<tr>
<td>6</td>
<td>3.40564</td>
<td>3.71631</td>
<td>3.90947</td>
<td>4.06501</td>
</tr>
<tr>
<td></td>
<td>(0.215558)</td>
<td>(0.270477)</td>
<td>(0.289772)</td>
<td>(0.328204)</td>
</tr>
</tbody>
</table>
Figure 1  Four stages of decision making by firms in period $t$

Figure 2  Sequence of R&D decisions within stage 2
Figure 3  Time series of endogenous turnovers from a single replication. a Number of entrants, b Number of exiting firms, c Total number of firms
Figure 4  Mean time series of endogenous turnovers from 500 independent replications. a Number of entrants, b Number of exiting firms, c Total number of firms
Figure 5  Mean time series of endogenous performance variables from 500 independent replications. a Market price, b Industry marginal cost, c Industry price-cost margin, d Aggregate profit
Figure 6  Time series of technological diversity and market share inequality (Gini coefficient). a Technological diversity: time series from a single replication, b Technological diversity: mean time series from 500 independent replications, c Market share inequality: time series from a single replication, d Market share inequality: mean time series from 500 independent replications
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**Figure 8**  Entries and exits in US automobile industry between 1895 and 1966 [Data: Smith (1968)]
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(b) Total number of exits
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[Data: Dunne, Roberts, and Samuelson (1988)]
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Figure 21  Histograms from the 500 independent replications, with and without endogenous R&D. a Rate of entry, b Rate of exit, c Duration of industry leadership, d Frequency of leadership changes
Figure 22  Impact of market size ($s$) and fixed cost ($f$) on R&D. a Aggregate R&D spending, b Share of innovation in aggregate R&D spending
Figure 23  Impact of market size ($s$) and fixed cost ($f$) on industry structure. a  Number of firms, b  Industry concentration
Figure 24  Impact of market size ($s$) and fixed cost ($f$) on industry performance.  
(a) Market price,  
(b) Industry marginal cost,  
(c) Industry price-cost margin
Figure 25  Steady-state mean technological diversity: Histograms from the 500 independent replications, with and without endogenous R&D
Figure 26  Impact of R&D costs, $K_{IN}$ and $K_{IM}$:  

- Steady-state mean volume of R&D, 
- Steady-state mean technological diversity
Figure 27  Impact of market size \((s)\) and fixed cost \((f)\) on the steady-state mean technological diversity

Figure 28  Steady-state mean market share inequality: Histograms from the 500 independent replications, with and without endogenous R&D
Figure 29  Impact of R&D costs, $K_{IN}$ and $K_{IM}$, on steady-state mean market share inequality

Figure 30  Impact of market size ($s$) and fixed cost ($f$) on the steady-state mean market share inequality
Figure 31  Distribution of ages of exiting firms in US auto industry (1895 – 1966). a Linear graph, b Log-log graph
Figure 32  Best fit for the distribution of ages of exiting firms in US auto industry (1895 – 1966)
Figure 33  Distribution of ages of exiting firms from the computational model (bin size = 1). a Linear graph, b Log-log graph
Figure 34  Distribution of ages of exiting firms from the computational model (bin size = 200). a Linear graph, b Log-log graph
Figure 35  Best fit for the distribution of ages of exiting firms from the computational model (bin size = 200)

Figure 36  Impact of endogenous R&D on the distribution of ages of exiting firms from the computational model (bin size = 200)
Figure 37  Proportion of firms exiting the industry that were of a given age (AGE) or younger.  

- **a** Impact of fixed cost \( (f) \),  
- **b** Impact of market size \( (s) \)