Inventories, Business Cycles, and Variable Capital Utilization*

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Abstract

Capital utilization varies over the business cycle. This paper integrates this fact with the (S,s) inventory model of Khan and Thomas (2007b). A model with variable capital utilization displays different dynamics than a model with fixed capital utilization. The volatility of net inventory investment is significantly higher and inventory/sales ratios are less countercyclical when utilization is variable. Many correlations more closely match the data in a model with variable utilization than in a model with variable utilization. Entrepreneurs clearly prefer variable utilization, given the choice. The results presented here suggest that they actually take steps to ensure it.

Keywords: Inventory, Capital Utilization, Business Cycles

JEL codes: E22, E32

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1 Introduction

In a recession, available resources, such as capital, are less fully utilized than in a boom. Plants that run multiple shifts and overtime during a boom decrease the number of hours that they operate (Shapiro 1993; Bresnahan and Ramey 1993, 1994; Hornstein 2002; Mattey and Strongin 1997). In the public's mind, one of the obvious traits about an economic downturn is that so many resources are going unused - workers are unable to find jobs, factories lie idle, and storefronts remain empty. One of the main purposes of Keynes's "General Theory" was to explain why resources may sometimes be underutilized.

Apart from casual observation, there is a strong reason to believe that, other things equal, optimizing agents would prefer processes that allow for variable utilization of factors. The additional flexibility of variable utilization provides agents with strictly more options when they must deal with shocks. If we believe that agents are aware that we live in a world of uncertainty, we shouldn't be surprised that they prepare for that uncertainty by choosing production techniques that allow them to vary their utilization of resources like capital.

However, in some modern macroeconomic theorizing, we move away from considerations about resource utilization. For example, it is not unusual for models to ignore the possibility of variable capital utilization (as in the inventory models of Khan and Thomas (2007b,a); Wen (2009); Wang and Wen (2009); Wen (2008)). In some models, capital does not appear at all (as in the inventory models of Kryvtsov and Midrigan (2009, 2010)).[1] While, at times, leaving out variable utilization can be a useful simplifying assumption, at other times, it significantly changes the dynamics or results of the model (Basu 1996; Basu and Kimball 1997; Shapiro 1993; Shapiro et al. 1996; Kydland and Prescott 1991; Lucas, Jr. 1970; Trupkin 2008). For example, Basu and Kimball (1997) notes that variable utilization interacts

[1]Often, this decision can be justified from the fact that in many aggregate production function estimates, capital appears with the wrong sign or not at all. (Burnside et al. 1995)

with imperfect price flexibility. In addition, Trupkin (2008) finds that in his inventory model, variable rates of capital and labor utilization impact the behavior of inventories over the business cycle.

Another case where replacing a fixed utilization with a chosen, variable utilization makes a significant difference in the model's performance is in a (S,s) inventory model styled after that developed by Khan and Thomas (2007b). Khan and Thomas (2007b) note that their model with fixed capital utilization has difficulty matching some key business cycle moments. To match these moments, Khan and Thomas (2007b) consider modifications to their model, but do not consider exchanging fixed capital utilization for variable capital utilization. Such an exchange does help move the model in the right direction - though it is an insufficient change to make the model match the data exactly.

In their paper, Khan and Thomas (2007b) explore an (S,s) inventory model with the goal of answering two questions. First, why are inventories pro-cyclical? Second, why is production more volatile than sales? In order to answer these questions, they suggest a model in which delivery of an intermediate good (which is combined with labor in producing the final product) is costly, as delivery requires labor hours. If there is a persistent positive productivity shock (a "boom"), final sales will rise (due to the increase in permanent income), which leads to an increased need for purchasing intermediate goods as existing stocks get used up more quickly, leading to increases along the extensive margin. At the same time, the increased productivity in their model is located in the intermediate goods sector. Seeing as all sectors are competitive in their model, lower marginal costs from increased productivity results in lower prices for the intermediate good, which, in turn, leads to increases along the intensive margin. This suffices to explain why inventories are pro-cyclical. Inventories rise in the aggregate because more firms have to replenish their diminishing stocks of intermediate goods, and intermediate goods are temporarily cheap, leading to larger orders.

The answer to the second question follows directly from the fact that inventory investment is procyclical. The variance of the summation of two random variables is equal to the summation of the variances and twice the covariance. If the covariance is positive, then the variance of the summation is greater than the summation of the variances. In this case, production is the summation of final sales and net inventory investment, and the two covary positively. So, production will be more volatile than final sales, as the variance of production is the sum of the variances of final sales and net inventory investment plus twice the covariance between sales and inventory investment.

Khan and Thomas (2007b) provide a good start, as their model does successfully replicate the two empirical facts which they seek to explain. However, there are significant differences between their simulated results and the data. In particular, "(a) the relative price of inventories is too countercyclical; and (b) the relative variability in net inventory investment is somewhat weak." (Khan and Thomas 2007b) To correct for these differences, Khan and Thomas (2007b) consider two changes to their baseline model. In their baseline model, they have a single productivity shock that affects one stage of production. They modify this model by considering a version in which there are productivity shocks at both stages of production. The second modification involves increasing the share of intermediate goods used in final good production. Finally, they consider a model with both of these modifications. When combined, these two changes help correct the two differences mentioned above. However, one can also make significant progress toward correcting these two problems through changing just a single assumption: rather than assuming that capital utilization is fixed, assume that it is a choice variable. Changing this one assumption does not perform as well as the combined modifications that Khan and Thomas (2007b) use on some measures; however, it outperforms the baseline model on many measures, and more closely matches the data than even the modified model on the countercyclicality of intentory-sales ratios.

2 Model

2.1 Model Overview

The model is a dynamic stochastic general equilibrium model including homogeneous households which consume final goods and provide labor, a set of heterogeneous final-goods-producing firms which hold inventories of intermediate goods to economize on fixed delivery costs for these intermediate goods, and a set of homogeneous intermediate-goods-producing firms which hold capital stocks. This form was chosen for a number of reasons. As noted by Khan and Thomas (2007b), manufacturing inventories are far more cyclical than retail and wholesale inventories.[2] Within manufacturing inventories, intermediate inputs are twice the size of finished good inventories (see Ramey and West (1999)). A model involving inventory movements over the business cycle should reflect the nature of those inventories that are most important during the business cycle.[3]

Variable capital utilization is included for two reasons. First, a model with fixed capital utilization implicitly assumes that either capital utilization does not vary, or that variations in capital utilization are unrelated to the subject at hand. However, examination of measures of capital utilization demonstrate that both assumptions are false. Capital utilization does vary over the course of the business cycle, and is procyclical (Shapiro et al. 1996; Hornstein 2002). Second, variable utilization has some intuitive hope of decreasing the countercyclicality of the relative price of inventories and of increasing the volatility of net inventory investment. Excessively countercyclical relative prices for intermediate goods should decline because

[2] Correlation between detrended inventories and GDP are 0.65 for manufacturing inventories, which is roughly twice the correlation for retail and wholesale inventories.

[3] This is the most significant difference between the model presented in this paper and that in Trupkin (2008). In Trupkin (2008), inventories are of final goods. In Khan and Thomas (2007b), as here, inventories are held because of a fixed cost of delivery of intermediate goods to final goods producers. In Trupkin (2008), inventories provide direct utility to households, as a proxy for decreased shopping time and increased variety.

when capital utilization is variable, marginal cost curves are less sensitive to productivity shocks. Therefore marginal costs will change less over the course of the business cycle. At the same time, with variable utilization marginal cost curves are somewhat less steep - and therefore the change in demand from the productivity boom will result in a relatively large change in the quantities of intermediate goods purchased. So, the relative price of inventories is less countercyclical, as the shift in marginal cost is smaller relative to the shift in demand. Also, net inventory investment is more volatile, as the shallower marginal cost curve results in large changes in quantities over the course of the cycle.

Following Khan and Thomas (2007b), markets are competitive, so that all agents take prices as given. This is done to show that imperfect competition is not necessary to reproduce the key business cycle facts that the model reproduces. (This is in answer to a challenge by Bils and Kahn (2000), which suggested that countercyclical inventory-sales ratios could not be produced in a competitive technology-driven business cycle model.)

2.2 The Aggregate State

In this model, there are three aggregate state variables. First, the exogenous technology process z follows a Markov chain process that is a discrete approximation[4] of the autoregressive process $\ln z' = \rho \ln z + \sigma_\epsilon \epsilon$ where ϵ is N(0,1). Second, a distribution of inventories μ is held by final goods firms. Finally, the aggregate capital stock K is composed of the capital stocks held by intermediate goods producers. The endogenous state vector (K, μ) is denoted as A, and the aggregate state is denoted as (z, A).

[4]The approximation is formed using the technique described in Tauchen (1987).

2.3 Agents

Households: The representative household receives a dividend from the profits of the firms, which depends on the aggregate state $D(z, A)$. In addition to this income, households receive wage income $\omega(z, A)n^h$ based on their choice of labor supply n^h. The household can also save using Arrow securities (though in equilibrium, household homogeneity will imply that these holdings will be zero). Households allocate current wealth from savings and income between consumption and Arrow securities (one security for each exogenous state) to maximize expected discounted lifetime utility. The household's problem can be written as:

$$h(a; z, A) = \max_{c, n^h, (a'_j)_{j=1}^{N_z}} (u(c, 1 - n^h) + \beta \sum_{j=1}^{N_z} \pi_{ij} h(a'_j; z_j, A')) \tag{1}$$

subject to

$$c + \sum_{j=1}^{N^z} Q_j(z, A)a'_j \leq a + D(z, A) + \omega(z, A)n^h \tag{2}$$

To rule out Ponzi schemes, the following constraints are imposed on household purchases of securities: $a'_j \geq \underline{a}$ where $\underline{a} < 0$. These constraints do not bind in equilibrium. The household takes as given the evolution of the aggregate state.

Final Goods Firms: The final-goods-producing firms make decisions in the following sequence: First, they decide whether or not to purchase intermediate goods in this period. Then, they decide how many of their intermediate goods (both those that were held in inventory and those that are newly purchased) and how much labor to use to produce final goods. This sequence of decisions gives rise to three sets of optimization questions that must be answered. First: given the aggregate state, what is the optimal level of inventories? The value of possessing that level of inventories will be denoted $v^a(z, A)$. Second: given that s is the level of inventories that the firm carried into this period, that the firm drew a level of delivery costs (ξ, measured in labor hours), and that the aggregate state is (z, A), is it optimal to adjust inventories to the optimal level, or to carry the current level of inventories into the production phase? The value of the optimal decision will be denoted $v^0(s, \xi; z, A)$. The final question is: given the level of inventories after the purchasing decision is made and the aggregate state, what is the optimal choice for labor input n and for inventory carried into next period s'? The value of the optimal decision will be denoted $v^1(s; z, A)$. The details for each value function are as follows:

$$v^a(z, A) = \max_s (v^1(s; z, A) - qs) \quad (3)$$

$$v^0(s, \xi; z, A) = qs + \max[v^1(s; z, A) - qs, v^a(z, A) - \omega\xi] \quad (4)$$

$$v^1(s; z, A) = \max_{n \geq 0, s \geq s' \geq 0} [G(s - s', n) - \sigma s' - \omega n + \sum_{j=1}^{N_z} Q_j \int_{\underline{\xi}}^{\bar{\xi}} v^0(s', \xi; z_j, A') h(\xi) d\xi] \quad (5)$$

The last term in 5 represents the expected value from carrying s' inventory units into the next period.

Intermediate Goods Firms: The difference between this paper's model and Khan and Thomas (2007b) is in the intermediate goods firms' problem. Given its capital stock, k, and the aggregate state, the representative intermediate goods firm chooses current employment l, next period's capital stock k', and current capital utilization u. Khan and Thomas (2007b) do not have variable capital utilization in their model. The intermediate goods firm's problem can be stated formally as:

$$w(k; z, A) = \max_{l \geq 0, k' \geq 0, u \geq 0} (qzF(uk, l) - \omega l - (k' - (1 - \delta(u)k) + \sum_{j=1}^{N_z} Q_j w(k'; z_j, A'))) \quad (6)$$

$\delta(u)$ should be increasing and convex to ensure an increasing marginal cost of capital utilization[5]. To provide a general form which satisfies these two restrictions, the following is chosen: $\delta(u) = \delta_0 u^{\delta_1}$.[6] For comparison, I also simulate a fixed

[5]These restrictions are present in Christiano et al. (2005) which finds $\delta(u)$ to be nearly linear, and also in Altig et al. (2005) which finds δ to be approximately cubic. The estimate of Justiniano and Primiceri (2008) suggests a much more convex function, with this coefficient in the neighborhood of 7 or 8.

[6]The more general form $\delta(u) = a + \delta_0 u^{\delta 1}$, as used in Trupkin (2008), which includes both wear-and-tear and rust-and-dust depreciation was considered, but calibrating such a form resulted in a degree of convexity that was not plausible, as it was far beyond the levels found in Christiano et al. (2005) or Altig et al. (2005). The chosen form has a precedent in Smith (1970).

capital utilization model.[7] The intermediate goods firms' problem is the same as that stated here, except that u is constrained to be exactly unity, and is therefore not a choice variable.

Equilibrium: Equilibrium is a set of value, choice, price, and state transition functions which satisfy the following conditions.

1. All agents optimize.
2. All markets clear.
3. The laws of motion for aggregate state variables are consistent with individual decisions.

Certain facts regarding equilibrium can be noted. First, because households are homogeneous, $a = 0$. Thus, household decisions regarding consumption and labor rely only upon the aggregate state.

As in Khan and Thomas (2007b), in equilibrium, the household's choice of Arrow securities implies that $Q_j(z, A) = \pi_{ij}\frac{\beta U_1(C(z_j,A'),1-N(z_j,A'))}{U_1(C(z,A),1-N(z,A))}$. Thus, the time varying discount factor can be substituted out of the firms' problems. By requiring that firms weight current profits by the household's marginal utility of consumption $p(z, A) = U_1(C(z, A), 1 - N(z, A))$, future profits will be discounted by the households' discount factor, β. Though this procedure will change the value functions, the decisions (which are the object of interest) will not change.

With this modification, the intermediate goods firms' problem becomes:

$$W(k; z, A) = \max_{l\geq 0, k'\geq 0, u\geq 0}(p(qzF(uk, l) - \omega l - (k' - (1 - \delta(u))k)) + \beta\sum_{j=1}^{N_z} W(k'; z_j, A'))) \tag{7}$$

This problem implies three first order conditions:

$$qzF_2(uk, l) = \omega \tag{8}$$

[7] This version is identical to the model in Khan and Thomas (2007b).

$$p(z, A) = \beta \sum_{j=1}^{N_z} W_1(k'; z_j, A') \tag{9}$$

$$qzF_1(uk, l) = \delta'(u) \tag{10}$$

Equation 8 equates the marginal revenue product of labor for the intermediate goods firms with its marginal cost. Equation 9 equates the expected marginal product of capital with the marginal product of consumption (capturing the savings-consumption tradeoff), and equation 10 equates the marginal revenue product of capital utilization with its marginal cost.

The first two first order conditions from this problem are the same as in Khan and Thomas (2007b); the last condition is a new restriction in this model. This condition implies that, if there is a positive technology shock (z increases), then u will probably increase (as long as z adjustments are not completely offset by changes in the price q), as the left hand side is decreasing in u, and the right hand side is increasing in u. This is consistent with common findings that capital utilization is procyclical.

Using the envelope theorem:

$$W_1(k; z, A) = p(qzF_1(u^*k, l^*)u^* + (1 - \delta(u^*))) \tag{11}$$

This can be substituted into equation 9 to give an equation which can be used to calculate current p:

$$p(z, A) = \beta \sum_{j=1}^{N_z} p(z_j, A')(q(z_j, A')z_jF_1(u^*k', l^*)u^* + (1 - \delta(u^*))) \tag{12}$$

Where u^* and l^* indicate the chosen optimum utilization and labor given the aggregate state in the next period.

3 Numerical Techniques

Solving the model in this paper requires a number of numerical solution techniques. The large state space must be simplified, the value functions of the final goods firms must be approximated (as they have a discrete decision), and expectations functions are difficult to derive analytically.

3.1 State Space Simplification

The first difficulty in solving the problem is the large state space. In particular, expectations about the future state depend on the distribution of inventories, which is a complicated variable. To solve this problem two steps are taken. First, the distribution of inventory holdings each period has been approximated by a discrete distribution with many nodes[8]. Second, following Krusell and Smith Jr. (1997a,b), the agents are assumed to only be concerned with a small set of moments of the distribution when forming expectations. In the present case (as in Khan and Thomas (2007b)'s), agents only consider the mean level of inventories.

3.2 Final Goods Firms

The most difficult part of the problem is to solve the final goods producing firms' value functions. To do so, I follow Khan and Thomas (2007b) in converting the final goods producing firms' value functions into the modified form (eliminating the time varying discount factor). The result is a set of three value functions:

$$V^a(z,A) = \max_s (V^1(s;z,A) - pqs) \tag{13}$$

$$V^0(s,\xi,z,A) = pqs + \max[V^1(s;z,A) - pqs, V^a(z,A) - p\omega\xi] \tag{14}$$

$$V^1(s;z,A) = \max_{n\geq 0, s\geq s'\geq 0} [p(G(s-s',n) - \sigma s' - \omega n) + \beta \sum_{j=1}^{N_z} \int_{\underline{\xi}}^{\bar{\xi}} V^0(s',\xi;z_j,A')h(\xi)d\xi] \tag{15}$$

[8] I used 2501 nodes spread evenly between 0 and 2.5, which span the range that inventories take in simulations.

These equations are solved on a grid using function iteration, and then splines are used to approximate the full functions. These approximations are used in simulations.

3.3 Expectations

One final problem was agents' expectations functions. Due to the complexity of the model, rational expectations are difficult to derive analytically. So, agents are assumed to form expectations using the following form: $x(z, A) = e^{\beta_0(z)} K^{\beta_1(z)} m_1^{\beta_2(z)}$ where m_1 is the mean level of inventories and x is p, q, K', or m_1'. The coefficients are dependent on the exogenous state z, and are reported in Tables 3 and 4.

3.4 Solution Procedure

The model is solved iteratively. Based on an initial guess for expectations, the value functions are approximated. Then, using the approximated value functions as a basis for firms' decisions, the model is simulated for 10,050 periods, starting from the steady state. The first 50 periods are discarded to ensure that the results are not sensitive to starting conditions. After that simulation, new expectations functions are calculated using the simulated data, and the value functions are reapproximated. The process continues until the path of the capital stock converges.

3.5 Equilibrium

To solve for equilibrium, I calculate p from equation 12. Then, I bisect to find the equilibrium intermediate good price q. Once p and q are found, the household's first order conditions will determine the equilibrium wage ω. Equation 12 ensures equilibrium in the final goods market, bisection for q guarantees equilibrium in the intermediate goods market, and the budget constraints and efficiency conditions guarantee labor market equilibrium.

3.6 Calibration

The model is calibrated so that $u^{ss} = 1$. Thus, the steady state for the model with variable capital utilization and the model with fixed capital utilization (where $u = 1$ by assumption) will be the same. Because this is the case, most of my calibration will follow Khan and Thomas (2007b), as their calibration was to match the steady state of their model (or their benchmark no inventories model) to certain key macroeconomic moments. The capital share α is chosen to reproduce the long-run business capital-to output ratio. The steady state depreciation rate is the average growth-adjusted ratio of investment to business capital. The intermediate good share in final good production is derived using input-output data from manufacturing and trade. The labor share in final goods production is chosen to imply a labor's share of output averaging 0.64. Preference parameters are chosen to give a real interest rate of 6.5 percent, and so that work-time is one-third of the time endowment. The stochastic productivity process's parameters were determined by the Crucini Residual approach described in King and Rebelo (1999). The fixed cost of inventory adjustment is uniformly distributed, with the lower bound set at zero. The upper bound and the cost of carrying inventories are calibrated so that, in the steady state, the cost of carrying inventories is 12% of output, and the inventory-sales ratio is 0.75. The only new parameter that must be calibrated in my model is the parameter for the convexity of the depreciation function. This was calibrated to normalize $u^{ss} = 1$. This calibration results in a function that is nearly quadratic, which is consistent with the assumption of Smith (1970) and Trupkin (2008), and falls between the estimates of Altig et al. (2005) and Christiano et al. (2005). Calibrated parameters are in Table 1.

4 Results

Results are reported in Tables 2-5 at the end of the paper.

In Table 2, the R^2-type measure of expectation function approximation is reported. The R^2 level for the variable utilization version suggests that there is some reasonably small error in the expectations functions for the mean level of inventories and the relative price of intermediate goods.

In Table 5, the ability of the two models to match certain business cycle moments is considered. In this table, the simulated series were HP-filtered, which makes these results analogous to those reported by Khan and Thomas (2007b). The first major result is that making capital utilization variable increases the relative standard deviation of net inventory investment. The relative standard deviation of the inventory/final sales ratio is lower in the variable capital utilization model. When compared to the data, the variable utilization model overcorrected these two moments. Looking at contemporaneous correlations, the variable utilization model is closer to the data in each case. In the fixed capital utilization model, the I/S ratio was too countercyclical; in the variable capital utilization model, the I/S ratio is still too countercyclical, but is closer to the data.

Table 5 reports some facts comparing the two models. The relative price of the intermediate good acts similarly in the two models. The mean level of inventories is less volatile when capital utilization is variable. The capital stock and capital services are less volatile in the variable utilization model than in the fixed utilization model. Consumption acts similarly between the two models, and labor supply is more volatile in the model with variable capacity utilization.

Table 5 focuses on some of the other key moments that the variable utilization model was designed to correct. With only two exceptions, the variable utilization model outperforms the fixed utilization model. As expected, the relative standard deviation of net inventory investment increased, and the relative price of intermediate goods is less countercyclical when capital utilization is variable. The only two points where the variable utilization model underperforms is: first, in the overcorrection of the volatility of net inventory investment; and second, in the weaker volatility of the relative price of intermediate goods.

5 Discussion

In Khan and Thomas (2007b), the authors note that one way in which they can increase net inventory investment is by decreasing the capital share. When the capital share is smaller, the marginal cost of intermediate goods falls more rapidly when there is a positive productivity shock. As a result, net inventory investment is far more volatile when they decrease the capital share. On this measure, introducing variable capital utilization has a similar impact. Net inventory investment is more volatile when capital utilization is variable. Rather than having just labor as a variable factor of production, the intermediate goods firms now have labor and capital utilization. So, the share of inputs that are variable in the short run has increased. However, the primary reason that net inventory investment is more volatile in the variable utilization model is not because marginal cost reacts more quickly to productivity shocks. Rather, variable utilization makes the supply of intermediate goods significantly more price elastic[9], and changes in demand for intermediate goods will have a larger impact on net inventory investment (as incentives on the final goods side have changed very little). Formally, the price elasticity of intermediate goods supply can be expressed as $\eta_m = \frac{1-\alpha}{\alpha}$ in the fixed capital utilization model (given our parameter values, this comes to about 1.67). In the variable capital utilization model the expression is $\eta_m = \frac{-\alpha-(1-\alpha)\delta_1}{\alpha(1-\delta_1)}$ (given parameter values, this comes to about 4.47). Thus, we would expect to see a higher volatility of inventory investment in the variable capital utilization model.

Though the volatility of inventory investment is significantly higher in the variable utilization model, the volatility of average inventory levels is only slightly higher. In a steady state, firms know that the price of intermediate goods is going

[9]Khan and Thomas (2007b) notes that in their model, a lower capital share flattens the marginal productivity of labor curve. This is equivalent to flattening the marginal cost curve.

to be stable for all of history. As a result, the share of firms adjusting inventories at each inventory level is the same each period, and the average level of inventories does not change. When aggregate uncertainty is included in the model, firms have an incentive to adjust their inventory holdings upward or downward based on the price of intermediate goods compared to the expected prices in the future. When supply is more elastic, there is less uncertainty regarding the price of intermediate goods in the future. This is reflected in the fact that the relative standard deviation of the prices of intermediate goods is smaller in the variable utilization model. Adding to this effect is the fact that marginal cost is less sensitive to productivity shocks in the model with variable utilization. In the model with fixed utilization, the elasticity of marginal cost with respect to productivity is $\frac{-1}{1-\alpha}$. In the parameterization used here, that comes to -1.597. When utilization is variable, the elasticity becomes $\frac{-\delta_1}{\alpha+\delta_1-\alpha\delta_1}$.[10] In the parameterization used here, that is -1.224.

The capital stock is less volatile in the variable utilization model. Variation in capital services occurs primarily through variation in utilization, and capital services are more volatile in the variable utilization model. As capital services are more volatile, so are the household variables of consumption and labor. Because capital services and labor are complements in the production function, then increased volatility of capital services led to an increased volatility of both labor input and the output of consumer goods. In terms of utility, households were somewhat better off under the variable utilization model. Over the 10,000 periods composing the simulation, the total utility[11] experienced by households was -19,026, while under the fixed utilization model total household utility was -19,089. This result is not surprising, as when you provide an optimizer with strictly more options, they certainly cannot be made worse off. However, this suggests that the additional volatility of consumption was offset by the fact that consumption was

[10] As δ_1 goes to infinity, this ratio goes toward the ratio for a fixed cost. That is because $\delta_1 = \inf$ is equivalent to the marginal cost of changing utilization being infinite - so that utilization is no longer variable. The same is true for the price elasticity of supply discussed in the previous paragraph.

[11] Here, simply a sum of each period utility.

higher - even if only slightly.

The expectations functions in Table 3 and 4 also provide an interesting comparison of the two models. The expectations functions for the future capital stock are similar - though it appears that the capital stock is slightly more persistent in the variable utilization model. The expectations functions for the mean future level of inventories is telling - the future level of inventories appear to depend less on the size of the current capital stock and less on the current level of inventories when capital utilization is variable. This reflects the additional flexibility provided by variable utilization. The marginal utility of consumption depends less on the level of the capital stock, and more on the mean level of inventories. Also, the price of intermediate goods depends more on the level of inventories - and is hardly impacted by the level of the capital stock at all. This fact gives further support to the idea that, when capital utilization is variable, the demand side of the market is more important in determining pricing than the supply side is - in our case, the supply side is less important because supply is more price-elastic and because marginal cost is less sensitive to changes in productivity.

6 Conclusion

Khan and Thomas (2007b) provide a foundation on which to build to explore facets of business cycle movements. I have built upon their foundation by extending their model to allow for variable capital utilization at the intermediate firms level. In doing so, we find that the dynamics of the model change significantly - and, along some measures (especially correlations), more closely match the data than in a similar model with fixed capital utilization.

In this model, we find that the dynamics change largely because variable capital utilization provides a degree of flexibility that is not present in fixed utilization models. As business cycle research moves forward, we must keep in mind that producers value flexibility and build it into their decision-making. All other things equal, entrepreneurs will choose a process with variable capital utilization. This paper shows that, when we take this flexibility into account, we can more correctly predict business cycle behavior.

Parameter	Value
β	0.984
η	2.128
α	0.374
θ_m	0.499
θ_n	0.328
δ_0	0.017
δ_1	1.9565
ρ	0.956
σ_ϵ	0.015
σ	0.012
$\bar{\xi}$	0.220

Table 1: Calibration of Parameters

Variable	Fixed CU	Variable CU
K'	1.0000	0.9990
m_1'	0.9998	0.9562
p	1.000	0.9902
q	0.9999	0.9670
N	10000	10000

Table 2: Expectation Function Accuracy

		z_1	z_2	z_3	z_4	z_5	z_6	z_7	z_8	z_9
K'	β_0	0.0770	0.0734	0.0759	0.0782	0.0814	0.0866	0.0950	0.0999	0.1041
	β_1	0.8867	0.8898	0.8876	0.8865	0.8849	0.8812	0.8755	0.8727	0.8703
	β_2	0.0317	0.0257	0.0242	0.0234	0.0234	0.0247	0.0292	0.0303	0.0308
m'	β_0	-0.3515	-0.3157	-0.3058	-0.3011	-0.2971	-0.2886	-0.3061	-0.3006	-0.3082
	β_1	0.1893	0.1702	0.1741	0.1778	0.1830	0.1854	0.2071	0.2094	0.2289
	β_2	0.6554	0.6758	0.6823	0.6816	0.6821	0.6876	0.6680	0.6680	0.6557
p	β_0	1.3725	1.3684	1.3621	1.3554	1.3480	1.3398	1.3312	1.3231	1.3167
	β_1	-0.3383	-0.3380	-0.3367	-0.3355	-0.3339	-0.3316	-0.3290	-0.3265	-0.3246
	β_2	-0.0488	-0.0469	-0.0458	-0.0450	-0.0446	-0.0448	-0.0454	-0.0457	-0.0454
q	β_0	-0.7861	-0.7881	-0.7944	-0.8016	-0.8135	-0.8132	-0.8242	-0.8328	-0.8508
	β_1	-0.1506	-0.1590	-0.1650	-0.1705	-0.1701	-0.1807	-0.1812	-0.1830	-0.1737
	β_2	-0.0664	-0.0616	-0.0597	-0.0584	-0.0595	-0.0509	-0.0517	-0.0494	-0.0540
N		367	761	1182	1801	2008	1742	1142	613	385

Table 3: Expectations Function Parameters - Fixed Utilization Model

		z_1	z_2	z_3	z_4	z_5	z_6	z_7	z_8	z_9
K'	β_0	0.0330	0.0404	0.0578	0.0691	0.0708	0.0679	0.0606	0.0559	0.0538
	β_1	0.9325	0.9309	0.9250	0.9213	0.9192	0.9160	0.9140	0.9139	0.9153
	β_2	-0.0025	0.0062	0.0272	0.0403	0.0386	0.0275	0.0093	-0.0020	-0.0062
m'	β_0	-0.3081	-0.3288	-0.3287	-0.3367	-0.3401	-0.3017	-0.2720	-0.2252	-0.2394
	β_1	0.0631	0.0817	0.0723	0.0756	0.0824	0.0824	0.0830	0.0825	0.1086
	β_2	0.5874	0.5524	0.5268	0.5011	0.4859	0.5362	0.5707	0.6350	0.6062
p	β_0	1.1125	1.0911	1.0575	1.0657	1.0894	1.0957	1.1010	1.1042	1.1207
	β_1	-0.1647	-0.1692	-0.1572	-0.1642	-0.1780	-0.1778	-0.1686	-0.1596	-0.1610
	β_2	-0.3149	-0.3482	-0.3866	-0.3747	-0.3428	-0.3258	-0.2991	-0.2749	-0.2354
q	β_0	-1.0239	-1.0662	-1.1448	-1.1594	-1.1227	-1.0972	-1.0646	-1.0489	-1.0233
	β_1	0.0122	0.0050	0.0262	0.0213	0.0016	0.0013	0.0065	0.0150	0.0081
	β_2	-0.3101	-0.3743	-0.4721	-0.4911	-0.4364	-0.3788	-0.2999	-0.2460	-0.1901
N		367	761	1182	1801	2008	1742	1142	613	385

Table 4: Expectations Function Parameters - Variable Utilization Model

Moment	US Data	Fixed CU	Variable CU
$\sigma(Y)$	2.237	1.904	2.434
$\sigma(FS)/\sigma(Y)$	0.710	0.840	0.733
$\sigma(I-S)/\sigma(Y)$	0.545	0.609	0.361
$\sigma(NII)/\sigma(Y)$	0.295	0.201	0.414
$\sigma(q)/\sigma(Y)$	0.653	0.509	0.495
$\sigma(c)/\sigma(Y)$		0.276	0.356
$\sigma(K)/\sigma(Y)$		0.370	0.210
$\sigma(L)/\sigma(Y)$		0.734	0.859
$\sigma(m_1)/\sigma(Y)$		0.758	0.822
$\sigma(uK)/\sigma(Y)$		0.370	0.686
Corr(FS,Y)	0.943	0.991	0.932
Corr(I-S,Y)	-0.381	-0.915	-0.784
Corr(NII,Y)	0.669	0.832	0.767
Corr(q,Y)	-0.257	-0.963	-0.816
Corr(NII,FS)	0.411	0.751	0.482
mean(c)		0.307	0.309
mean(K)		1.795	1.826
mean(L)		0.341	0.342
mean(m_1)		0.550	0.571
mean(q)		0.416	0.417
mean(uK)		1.795	1.822

Table 5: Model Comparison

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