Determining the Transients and Steady State Charging Current and Voltage for RL Circuit

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April 25, 2009

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Purpose

To develop time domain mathematical equation for charging current in a series RL circuit using Laplace Transform

- Develop of equations for transient current and voltage equations requires the use of differential equations (Time Domain)
- Typically beyond the mathematical capability of Freshmen engineering students
- Utilization of Laplace transformation (Frequency domain) would enable the solution to be obtained using only algebra

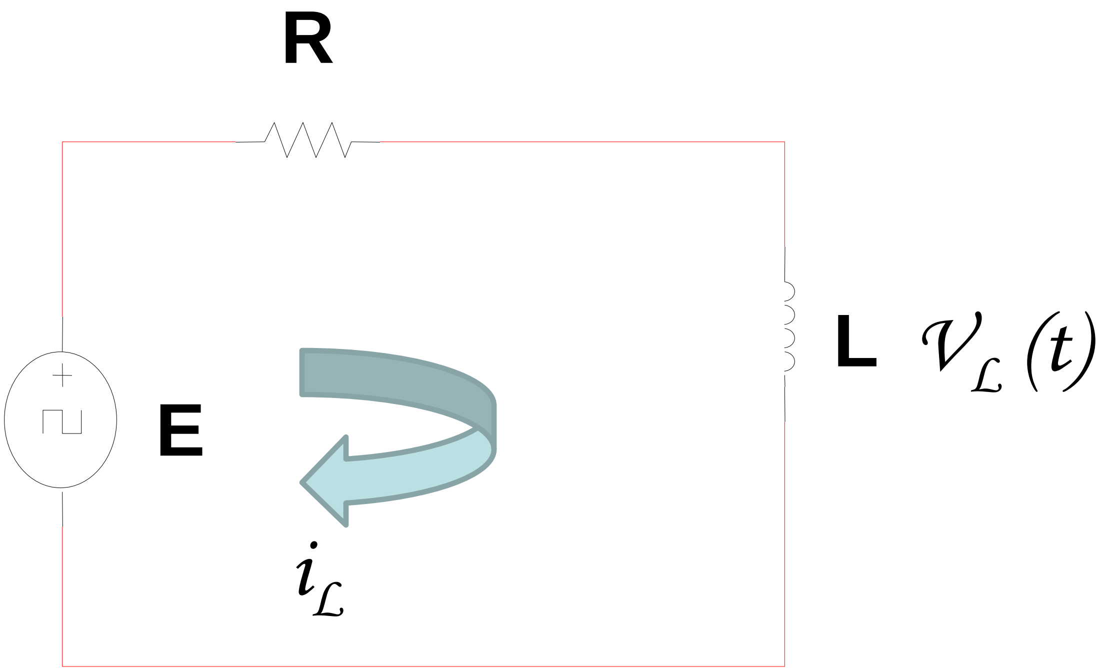
Terms and Definitions

- **Transients** –
  A short-lived oscillation/decay in a system caused by a sudden change of voltage or current expressed as a function of time exponentially
- **Inductor-**
  An electrical device (typically a conducting coil) that introduces inductance (stored current) into a circuit
- **Steady State-**
  A condition of a physical system or device that does not change over time
- **Laplace Transform-**
  A mathematical technique to enable solution of Differential Equations utilizing algebra via transformation between time and frequency domain

RL Circuit Elements

- **Voltage Source - E (Volts)**
- **Resistor - R (Ohms)**
- **Inductor – L (Henry)**
- **Inductor Current – I_L (Ampere)**
- **Inductor Voltage - V_L(Volts)**

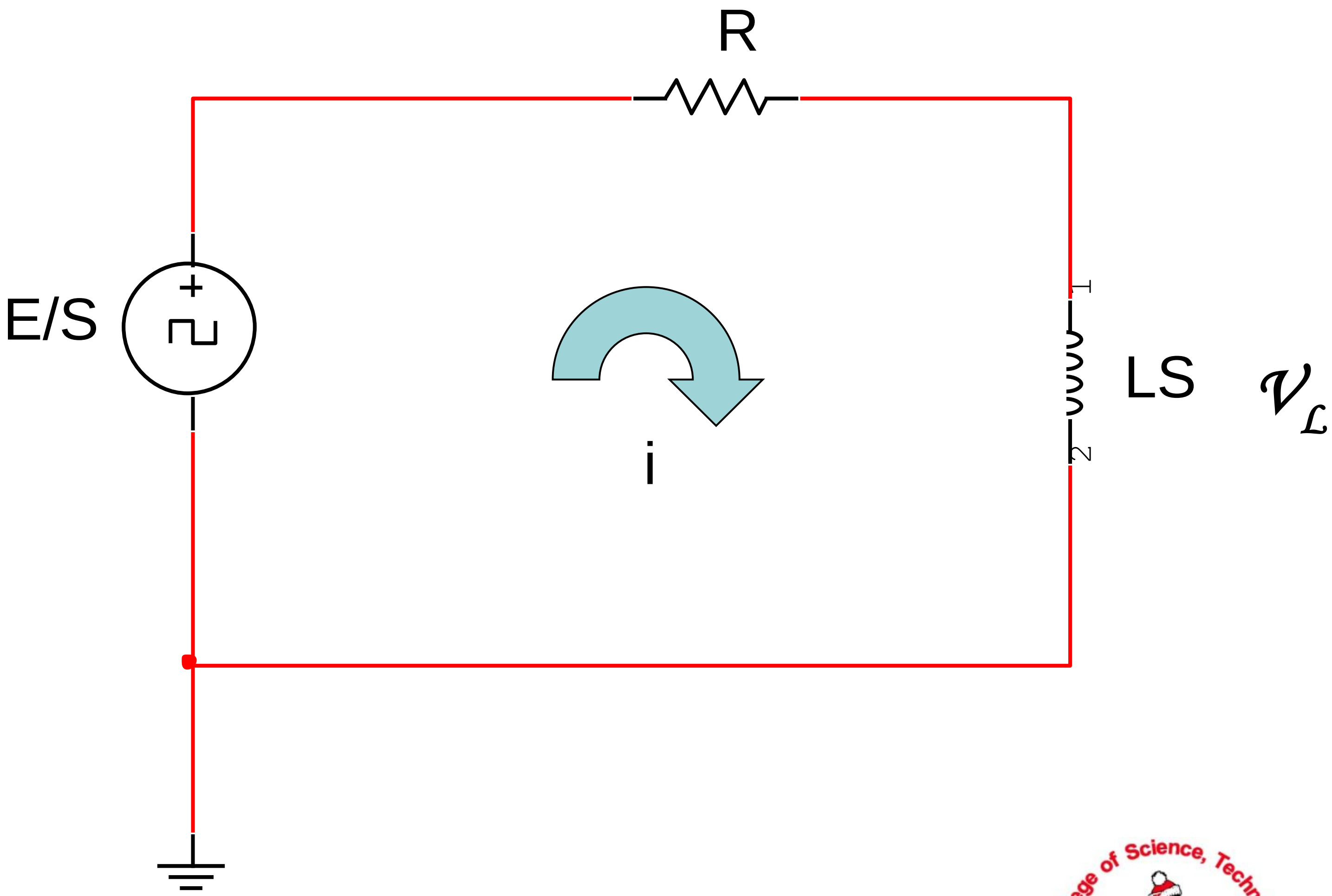
RL Circuit - Time Domain Differential Equation



$$iR + L\frac{di}{dt} = E$$

(Kirchhoff' Voltage Law)

Laplace Transform (Frequency Domain) Circuit



Transient Current Equation - Laplace Transform (Frequency Domain)

$$I(s) = \frac{\frac{E}{S}}{R + LS} = \frac{E}{(S)(R + LS)}$$

$$= \left(\frac{E}{L}\right)\frac{1}{S\left(S + \frac{1}{\tau}\right)}$$

Applying Partial Fraction

$$= \frac{E}{L}\left(\frac{A}{S} + \frac{B}{S + \frac{1}{\tau}}\right)$$

$$\text{Where } \tau = \frac{\text{L}}{\text{R}}$$

Transient Current Frequency/Time Domain Transformation

$$I\,(S) = \frac{E}{R}\left[\frac{1}{S} - \frac{1}{S + \frac{1}{\tau}}\right]$$

a = $^{1}/_{\tau}$

By applying inverse Laplace Transform Properties

$$\mathfrak{L}^{-1}\left(\frac{E}{S}\right) = E \qquad \mathfrak{L}^{-1}\left(\frac{1}{(S+a)}\right) = e^{-\frac{t}{\tau}}$$

The Frequency Domain equation can be transformed into Time Domain equation

$$\boldsymbol{i(t) = \frac{E}{R}\left(1 - e^{\frac{-t}{\tau}}\right)}$$

Charging Voltage - Frequency Domain Equation

Applying voltage divider, the inductor voltage is:

$$\mathcal{V}_{\mathcal{L}} = \frac{E}{S}\left(\frac{LS}{R+LS}\right)$$

After simplifications:

$$\mathcal{V}_{\mathcal{L}} = \frac{E}{S+\frac{R}{L}} = E\left(\frac{1}{S+{}^{1}/_{\tau}}\right)$$

Time constant **τ = L/R**

Transient Voltage Frequency/Time Domain Transformation

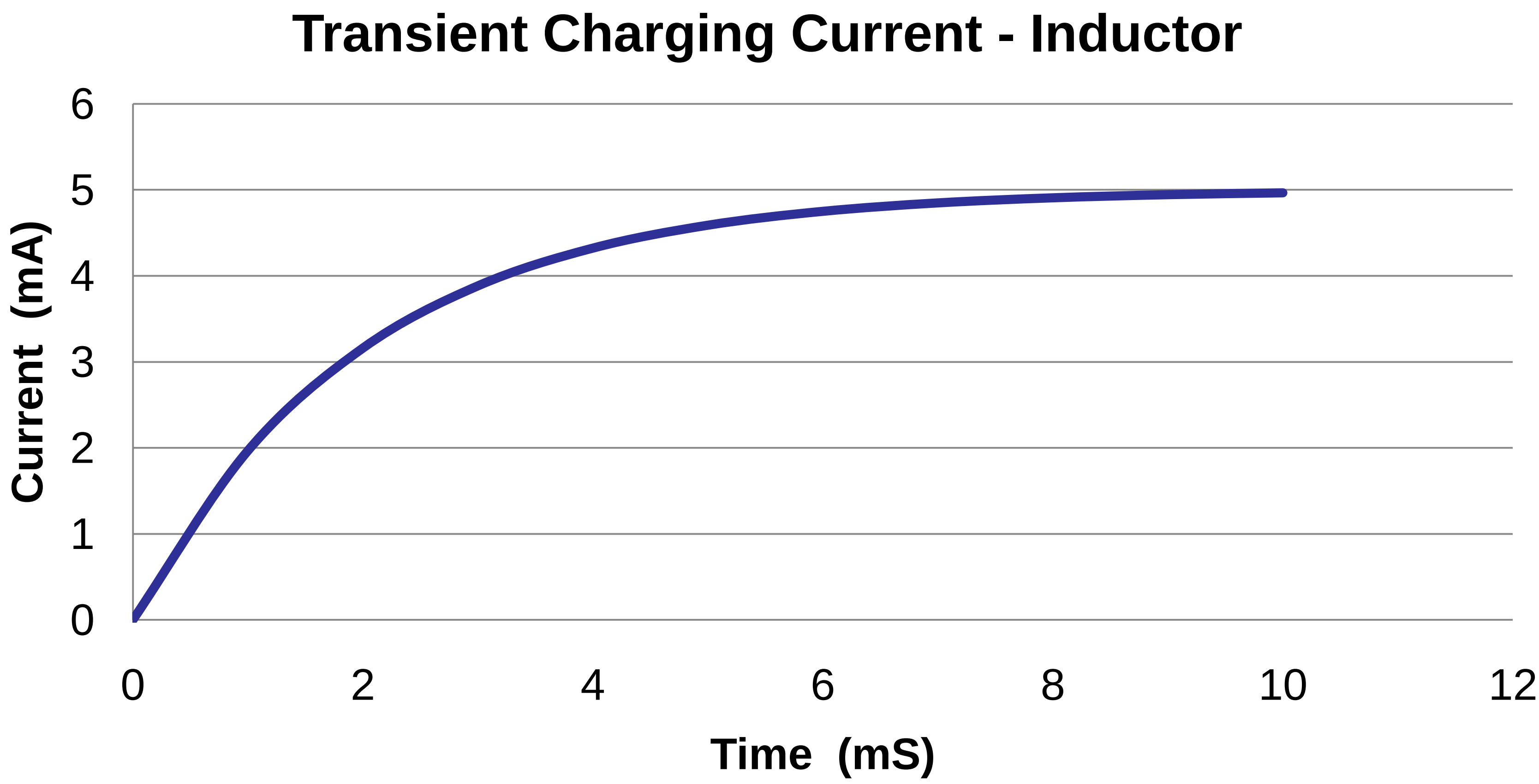
Applying Inverse Laplace Transform Property :

$$\mathfrak{L}^{-1}\left(\frac{1}{(S+a)}\right) = e^{-\frac{t}{\tau}} \qquad a = {}^{1}/_{\tau}$$

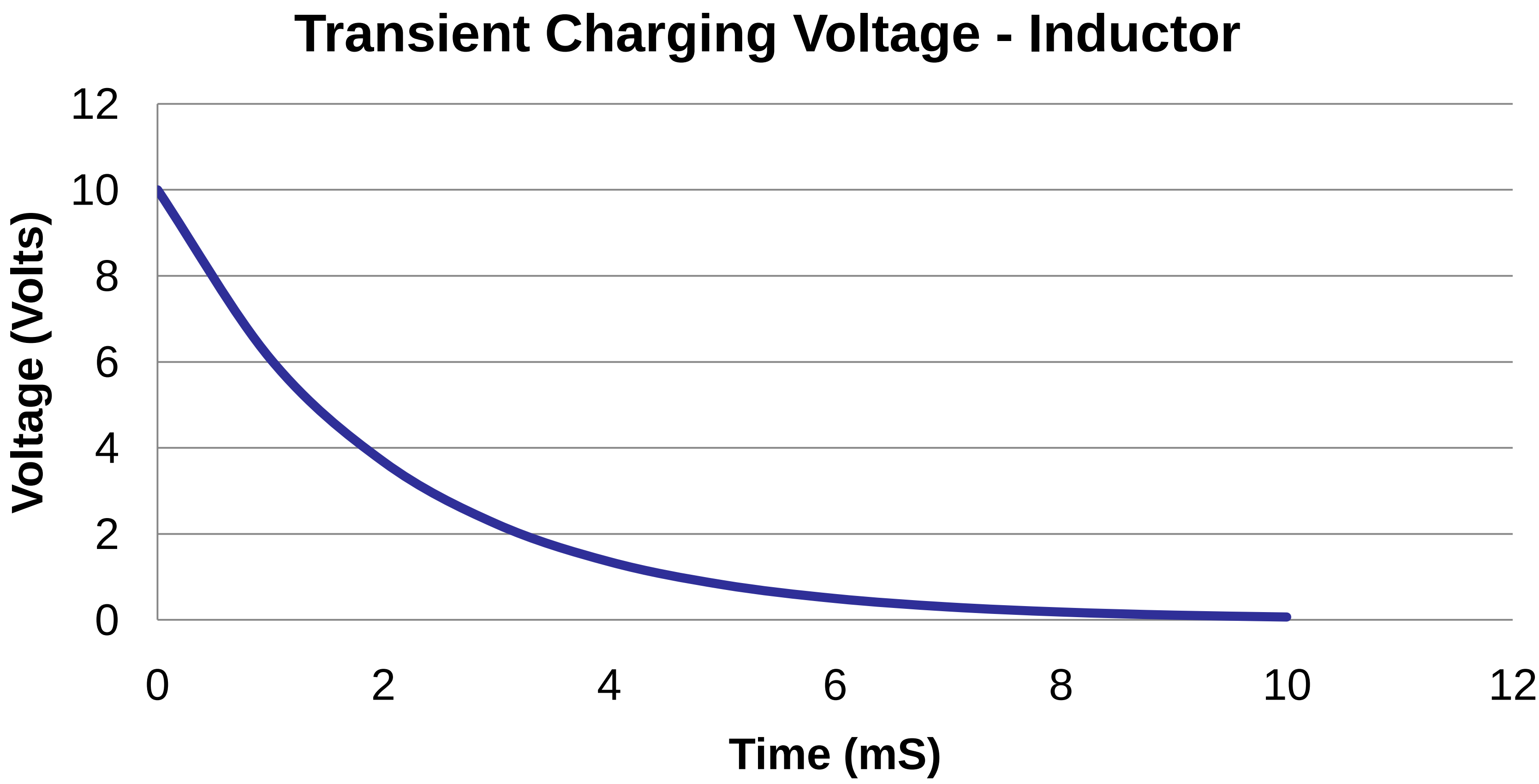
The time domain equation is

$$V_L = Ee^{-t/\tau}$$

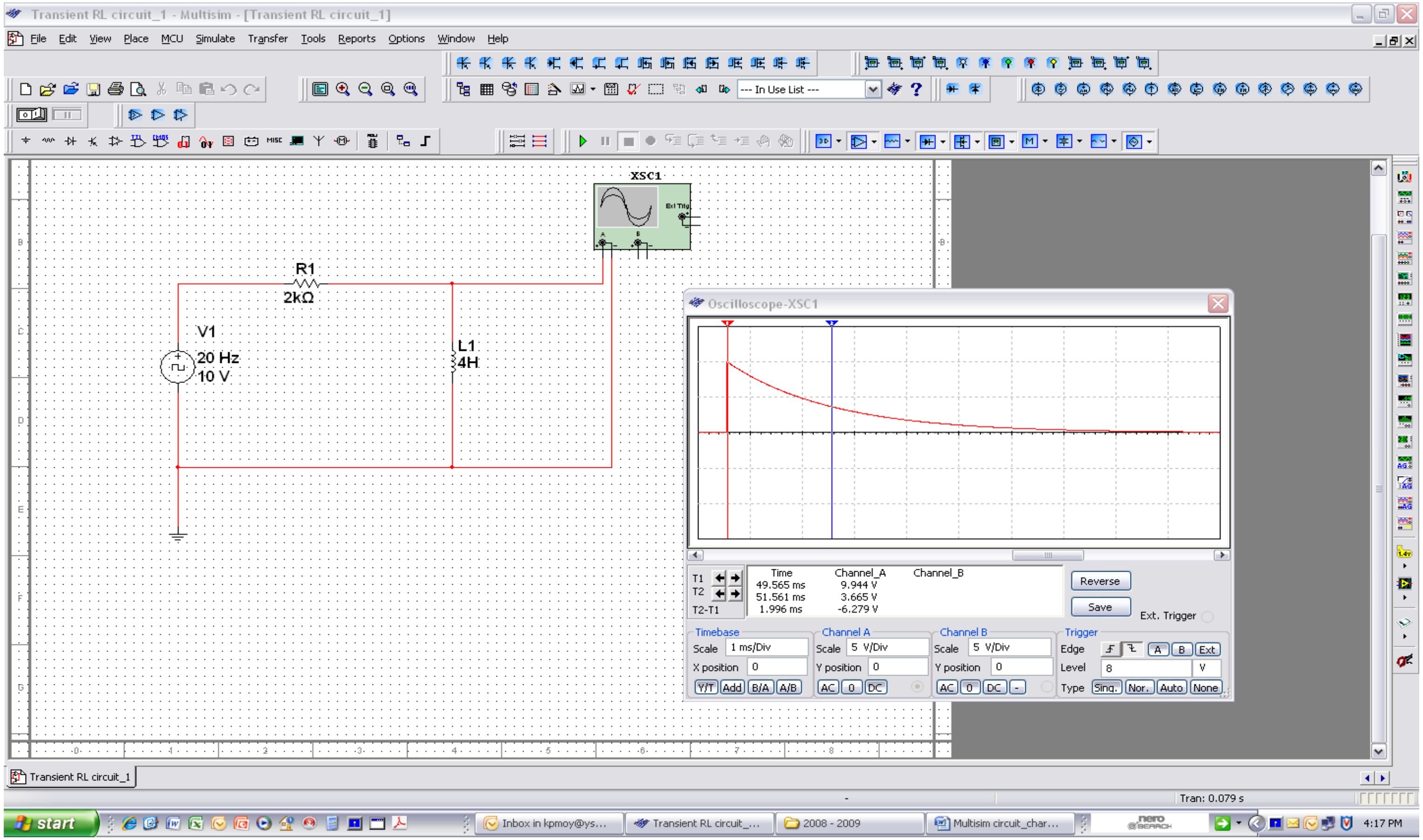
Transient Current Time Domain Waveform – Excel



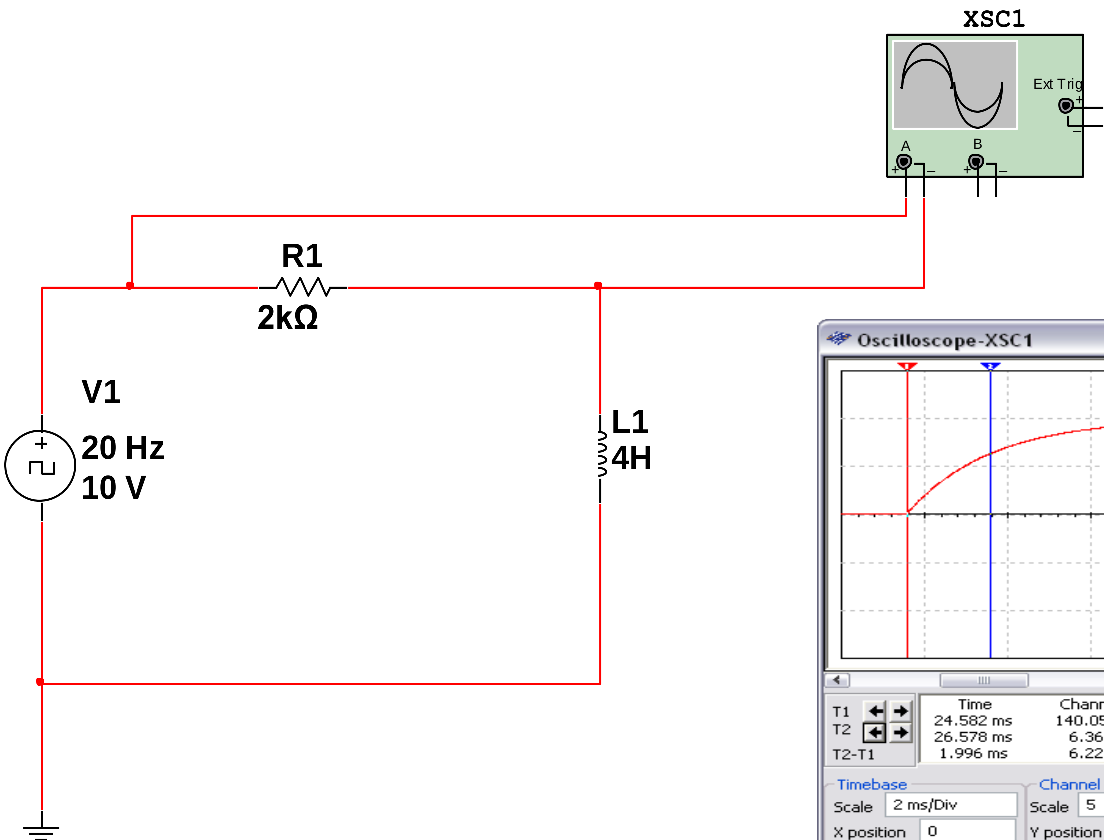
Transient Voltage Time Domain Waveform – Excel

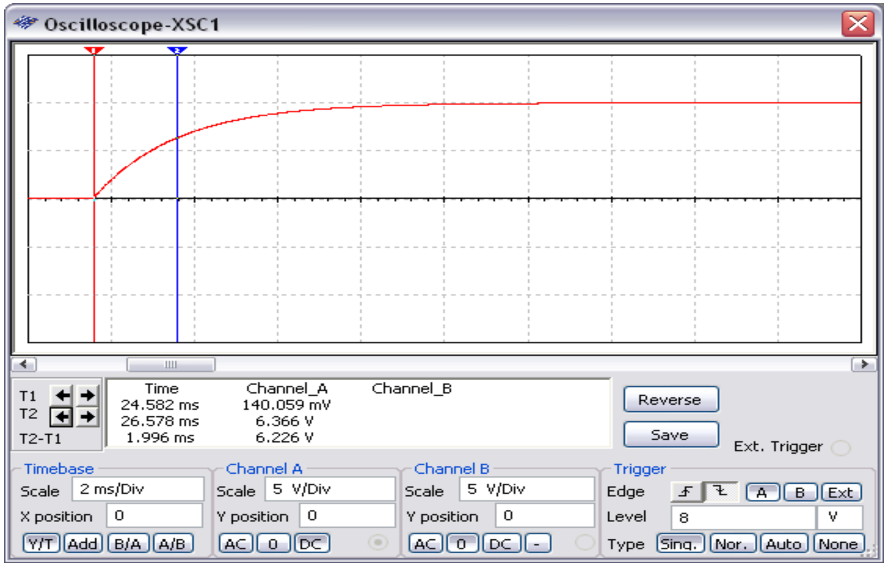


Transient Voltage Simulation (Time Domain)– Multisim

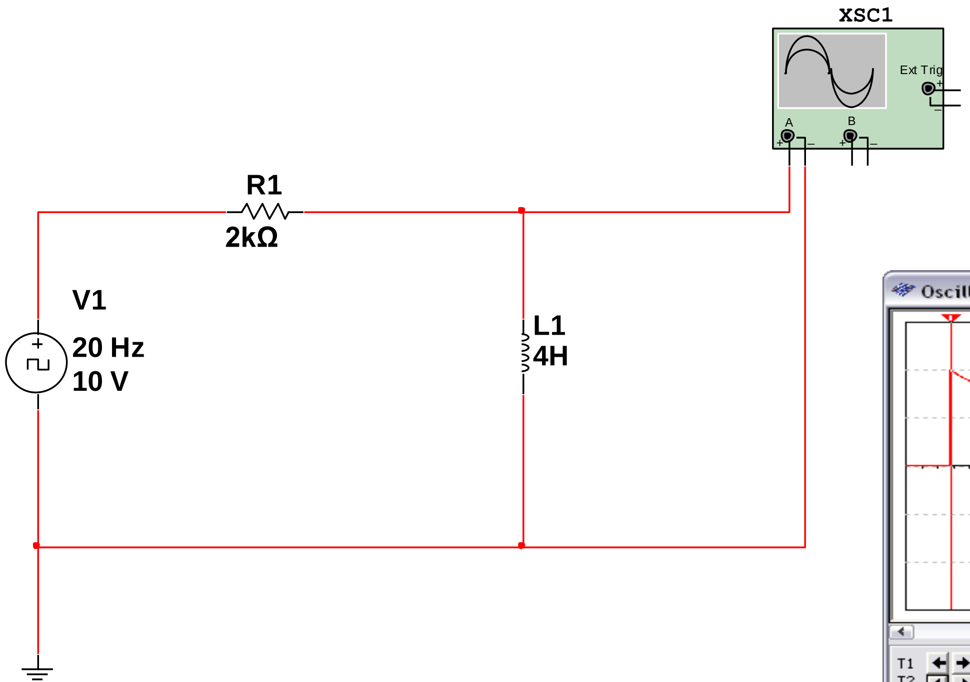


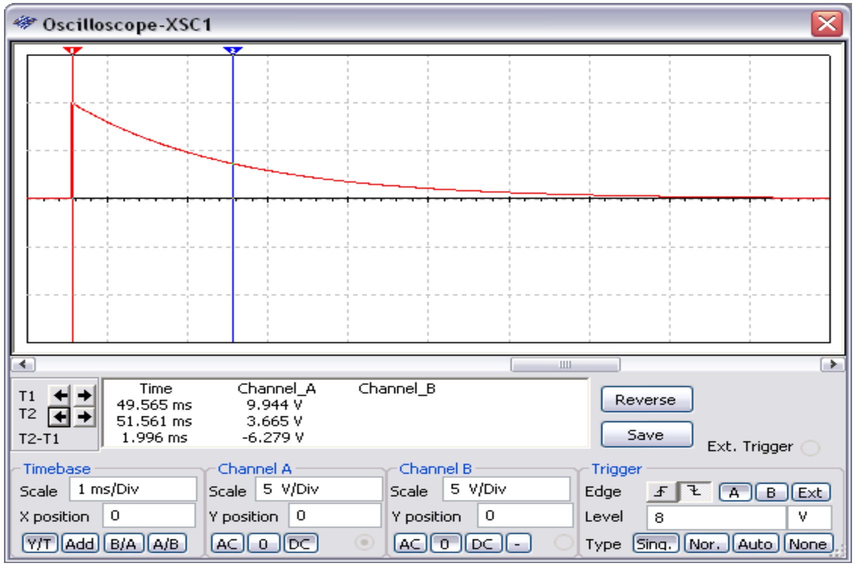
Transient Current Simulation (Time Domain)– Multisim





Transient Voltage Simulation (Time Domain)– Multisim





Calculation:

Tau = L/R = 4 H/2kΩ = 2 mS

Tau(calculated) = 2 mS

Tau (measured) = 1.996 mS

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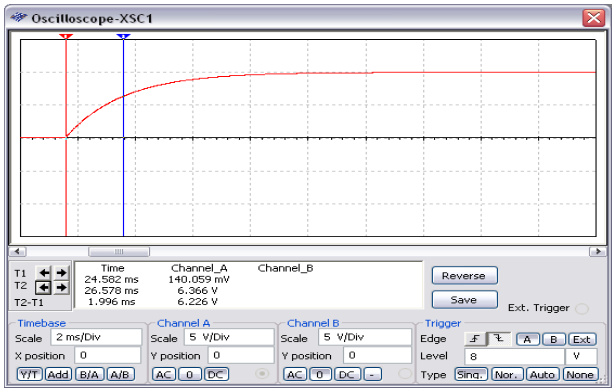
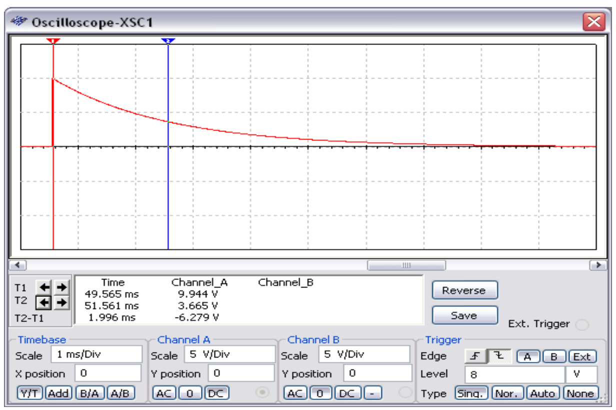
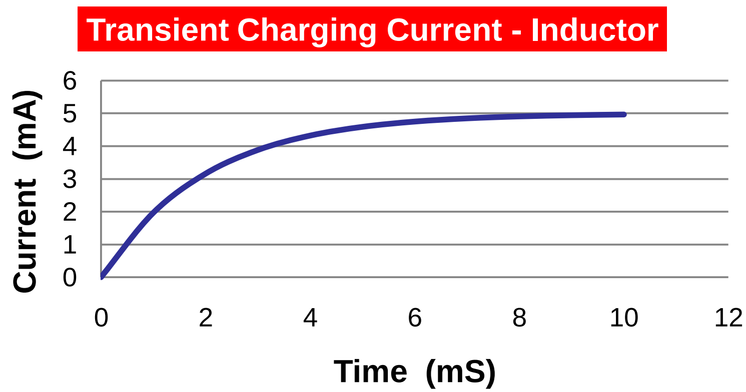
Comparisons between Multisim and Excel Graphs

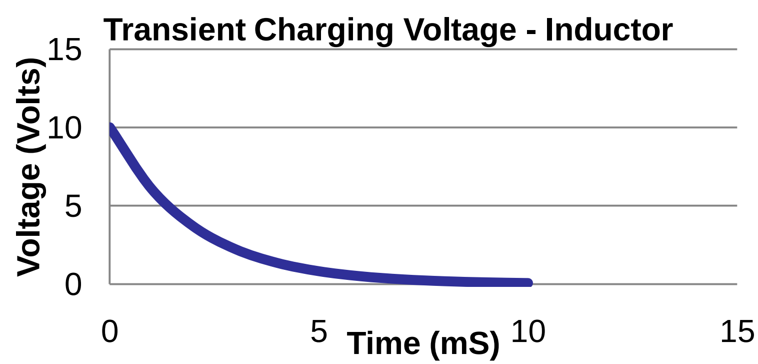
Multisim





Excel





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Summary

- Established RL circuit and develop differential equation for charging current and voltage for the inductor
- Utilized Laplace transform to change time domain differential equation into frequency domain equations
- Simplified equations using Partial Fraction
- Applied inverse Laplace Transform to covert equation back into time domain
- Developed current and voltage vs. time graph using Excel
- Performed circuit simulation (Multisim) to obtain time domain current and voltage graph
- Compare graphs developed by Multisim & Excel