1001011010100100010110011011010100011
1111110011001011010100100010110011011
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The Game of Nim
Winning Strategies Defined Through Binary Numbers

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ABSTRACT

Even a game as simple a the game of nim has a winning strategy that is defined mathematically. However, the simplicity of the game is not reflected in the complex mathematical definition of the game's strategy to win. It is through the simple yet complex use of binary numbers one recognizes winning and losing positions of the game of nim and ultimately the winning strategy.

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INTRODUCTION

With an uncertain origin, the game of nim has been around for centuries. The common name of nim is thought to be derived from the German word nim "to take," however, the game has various names. Even though there is an unknown origin the game has been known to be played in China, Africa, and Europe over the course of its very little documented history. There are two versions of the game both containing the same basic rules, but different objectives. The rules are simple,

Rules
Objects are arranged into rows, there can be any number of rows containing a various number of objects, in this example coins are used.
Two players take turns removing the coins from the rows.
Decided who goes first.
A player can take any number of coins he or she desires but only from one row.
A player must remove at least one of the coins during their turn.
Players continue to remove the coins until zero coins remain.
Objective:
To remove the last coin and win.

The following methods inform one on the winning strategy of the simple game. The data uses the objective of the player who removes the last coin is the winner. The strategy is explained in mathematical terms under the basis of binary numbers. Complex strategy of the game is surprising compared to its awe-inspiring simplicity.

METHODS

Winning scenarios:
1.If one coin is left, obviously, you win by taking the last coin.
2.If one row is left, you can take the whole row and win
3.If you are in neither of these situations, the objective is to maneuver your opponent into a losing position

Losing scenarios:
1.If there are two coins left, and they are in separate rows, your opponent can only take one coin, thus, allowing you to take the remaining coin. This is a losing position for your opponent.
2.If there are two coins remaining in one row and one coin remaining in the other row, your opponent will take one coin from the row with two coins, thus, putting you in the previously mentioned losing position. This is a losing scenario for you.

The reason that the first scenarios resulted in a win and the second scenarios resulted in a loss is the idea of balancing. The first scenario was a balanced position because both rows contained the same number of coins, one. The second scenario was an unbalanced position because one row contained two coins and the other only had one. When the coins are in a balanced position, the position is a losing one.
As an example lets take two rows of five coins each. Lets say that your opponent removes three coins from one of the rows. In order to return the coins to a balanced position, you would take three coins from the other row. This leaves two coins in each row. Your opponent now has two options. They can either take one coin or two coins from one of the rows. If they take one coin, you can balance it back out by taking one coin from the other row. This results in the losing position previously mentioned. If your opponent takes two coins, you are left with one row of two coins and you can take them both and win the game. A balanced position is always a losing position because it can always be changed into these two basic losing positions. For games with more than two rows, it is hard to calculate the balanced positions in your head. A way for fixing this problem is to use binary numbers.

RESULTS

Binary Numbers:
Binary numbers are converted from the well known decimal system of ten digits into ones and zeros. In order to convert from decimal to binary, the powers of two must be used.

Ex. Convert 10 into binary
The highest power of two that is less than ten is 2^3 or 8. We can, therefore, conclude that eight goes into ten one time. After subtracting this eight from ten, we are left with two. The next power of two is 2^2 or 4. Four goes into two zero times. We then proceed to the next power of two, 2^1 or 2. Two goes into two one time. We then are left with a remainder of zero. Lastly, we are left with 2^0 or 1. Zero goes into one zero times. Putting these numbers together we get the binary number of 1010.
10: (1)8 + (0)4 + (1)2 + (0)1
We can also do this process in reverse.

Ex. Convert the binary number 101 into decimal
The one on the far right corresponds to 2^0 or 1 and so on for the powers of two moving to the left. From the powers of two we get one, one; zero, twos; and one, four,
Adding these together we get five.
101: 1(1) + 0(2) + 1(4) = 5

In order for binary numbers to help us in the game of nim, we need to understand how to add and subtract them. Addition and subtraction work the same in binary as they do in decimal.

Ex. Add 101 to 1010
1010
+101
1111
We know that 1010 is equal to ten and 101 is equal to five. Add these together and you get fifteen. Fifteen represented in binary is 1111, so this works.

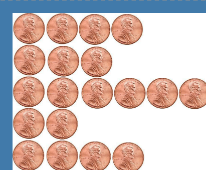

Figure 1. Possible Starting Position of Infinite Possibilities

$$23 = 16 + 7$$
$$23 = 16 + 4 + 3$$
$$23 = 16 + 4 + 2 + 1$$
$$23 = 2^4 + 2^2 + 2^1 + 2^0$$
$$23 = 1{*}2^4 + 0{*}2^3 + 1{*}2^2 + 1{*}2^1 + 1{*}2^0$$
$$23 = 1\;0\;1\;1\;1$$

Figure 2. Converting a Number to Binary

$$\underset{\text{Winning}}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}} \longrightarrow \underset{\text{Losing}}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}}$$

Figure 3. A Binary Matrix Representation of **Figure 1** on the left and a Possible Move to create **Figure 4.** Matrix on the Right

Figure 4. Removing Five From Row Three Makes the Matrix Balanced Therefore a Losing Position

RESULTS

Ex. Add 111 to 101
Adding two ones together results in a zero and a one is carried over just like in regular addition.
111
+ 101
1100
Subtraction works the same way with binary numbers.
Ex. Subtract 101 from 1010
1010
- 101
101
In the case of zero minus one, you carry a two over, which gives you
2-1 or 1 in the far right space. Because you did this, you have to subtract one from the next column leaving you, in this case, with zero
10-5 = 5 This subtraction works.
.
The Use of Binary in the Game of Nim:
As stated earlier, balanced positions are losing positions. The same holds true for binary numbers of the position. By converting the number of coins in each row to binary, we can check if the position is balanced or not. A position is balanced in binary if all of the columns add up to an even number.
Ex. Lets take a position that we know is balanced, 2 coins in each row.
010 Converting 2 to binary is 100.The columns add up to even
010 numbers (2,0,0). A more complex example contains three
rows. (7,5,2). By looking at the decimal numbers it is nearly impossible to tell that
111 the position is balanced. Through the use of binary numbers,
101 we can tell that the position is balanced because the columns
010 add up to even numbers. (2,2,2). If a position is unbalanced,
then we want to make it balanced. This is not a balanced position, but we can
111 use subtraction to balance it (3,2,2).
101 The first column adds up to three and we only want it to add
110 up to two. Therefore, we pick one of the rows to change. We'll
pick the last row and try to make it look like the previously mentioned balanced position. We have 110 in that row and we want 010. To find out what we must
110 remove in order to get there we subtract the two. The answer
-010 we get is 100 in binary. Convert this to decimal and 100
the answer is four. We must remove four coins from that row. This makes sense because we started with 110 (6 in decimal) and wanted to get to 010 (2 in decimal). To get from 6 to 2 we have to remove 4.

CONCLUSIONS

From the following data one can see how the ending surprise of who is going to win is taken away when two who know how to play challenge each other in the game of nim. The two challengers know their next move after the other player goes only waiting for them to make a mistake to ensure their victory. Masters of the game of nim know who will win upon the placement of the coins. Try to play and wait for you opponent to make a mistake.

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