Multiple Accelerations on a Distant Planet

Josh Orzel, Sean Sheridan, Peter Fumich, Candice Quinn
Advisor: Dr. Kiril Streletzsky, Physics Department.

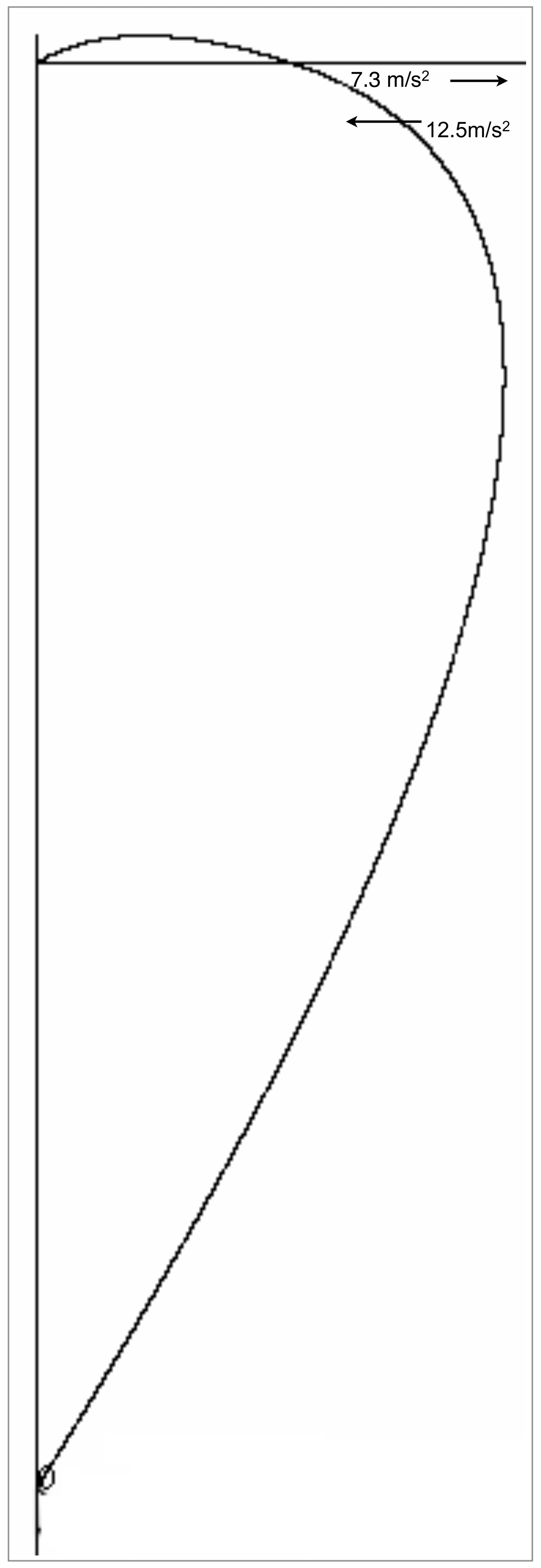
Introduction:

Kinematics is the study of how things move. Motion is described in terms of displacement, time, velocity, and acceleration. Velocity is the rate of change of displacement and the acceleration is the rate of change of velocity. Vertical motion under the influence of gravity can be described by the basic motion equations. Given the constant acceleration of gravity, the position and speed at any time can be calculated from the motion equations. In the project we will be dealing with multiple constant accelerations, constant gravity, and changing magnetic force effecting the horizontal component of velocity.

Abstract:

We are on the planet Newtonian. The Jetsons are on their yearly family vacation. Elroy is wondering around and finds a cliff that is very very tall (so tall that we assume it is infinitely tall). Elroy launches a metal ball off of the cliff at an angle of 32º toward the east with a velocity of 2.889 m/s. As he watches the ball go, he notices a VERY odd thing, there are multiple accelerations acting on it. He gets out his pocket computer and analyses the area. He finds out that the gravity on Newtonian is 10.33m/s². However there is a magnetic field acting in the area also. This magnetic field changes, depending if it is above the cliff or below the cliff; above the magnetic goes to the east with acceleration of 7.3m/s² and below the acceleration is 12.5m/s² to the west. How far below Elroy will the ball hit the cliff?

Path of flight



The path of flight was created using a java program which utilized the multiple accelerations and created a parametric curve, which fit the given parameters.

Approach

This is a trajectory problem solved using Kinematic Equations, which are able to be utilized for any motion which has either constant velocity or constant acceleration. The initial velocity given was split using methods from calculus.

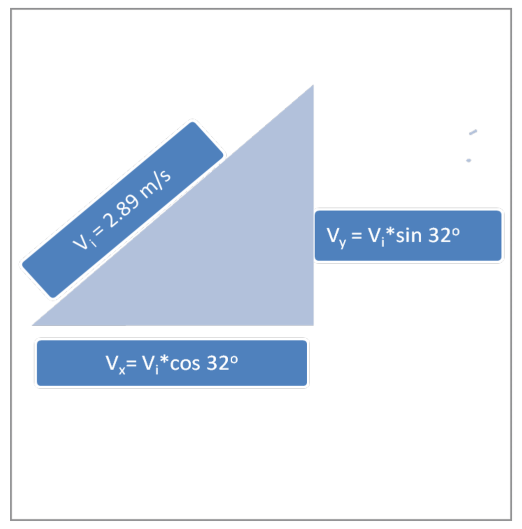
Initial horizontal and vertical velocities

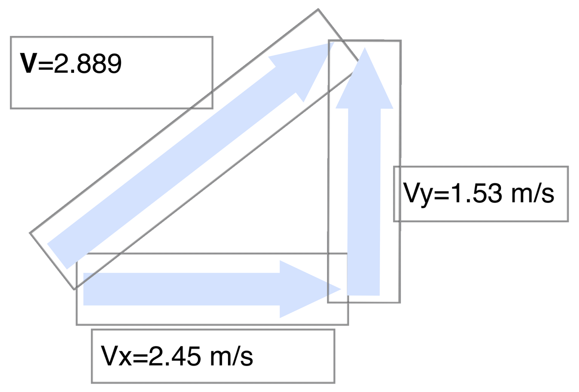
First taking the initial velocity and the angle of inclination we can find the velocity components in the horizontal and vertical directions.

$V_i = 2.889\,\frac{m}{s}$ $\theta = 32^o$

$$V_x = V_i \cdot \cos\theta = \left(2.889\,\frac{m}{s}\right)\cdot\cos(32^0) = 2.45\,\frac{m}{s}$$

$$V_y = V_i \cdot \sin\theta = \left(2.889\,\frac{m}{s}\right)\cdot\sin(32^0) = 1.53\,\frac{m}{s}$$





Maximum height of trajectory above the cliff:

Next we can find the maximum height by using the fact that the vertical velocity is 0m/s at the maximum height of the trajectory.

$$V_{yf}^2 = V_{vi}^2 + 2\,a_y\Delta y$$

$$0 = \left(1.53\,\frac{m}{s}\right)^2 + 2\left(-10.33\,\frac{m}{s^2}\right)\Delta y$$

$$\Delta y = \frac{V_{yf}^2 - V_{vi}^2}{2\,a_y}$$

$$\Delta y = \frac{0 - \left(1.53\,\frac{m}{s}\right)^2}{2\left(-10.33\,\frac{m}{s^2}\right)}$$

The maximum height the ball will reach: $\Delta y = 1.13\cdot 10^{-1} m$

Time above Cliff:

We can now find the time it takes to reach this height:

$$a_y = \frac{V_{yf} - V_{yi}}{t}$$

$$t = \frac{V_{yf} - V_{yi}}{a_y}$$

$$t = \frac{0 - 1.53\,\frac{m}{s}}{\left(-10.33\,\frac{m}{s^2}\right)}$$

$$t = 1.481\cdot 10^{-1} s$$

Now we can take this time times two for the total time the ball travels above the cliff:

$$time_{total\,above\,cliff} = 2\cdot t = 2\cdot 1.481\cdot 10^{-1} = 2.962\cdot 10^{-1} s$$

Maximum horizontal displacement:

Using this total time we can find the change in the horizontal direction:

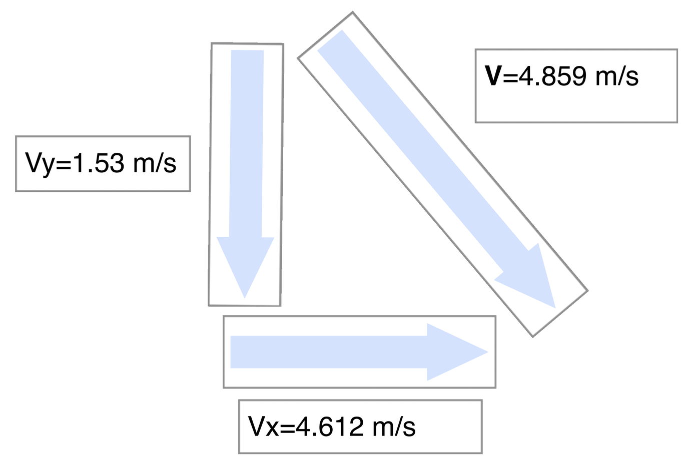
$$\Delta x = V_{xi}\cdot t + \frac{1}{2}a_x\cdot t^2$$

$$\Delta x = \left(2.45\,\frac{m}{s}\right)\cdot(0.2962\,s) + \frac{1}{2}\left(7.3\,\frac{m}{s^2}\right)\cdot(0.2962\,s)^2$$

The maximum horizontal displacement above the cliff:

$$\Delta x = 1.046\,m$$

Trajectory below the cliff:



First we need the final velocity from above the cliff, below the cliff the magnetic field is in the opposite direction affecting the horizontal velocity. Using the same method as above the cliff for finding the maximum height, we can find the maximum horizontal displacement using the final velocity from above as our new initial velocity:

$$V_{xf} = a_x\cdot t + V_{xi}$$

$$V_{xf} = \left(7.3\,\frac{m}{s^2}\right)\cdot(0.2962\,s) + 2.45\,\frac{m}{s}$$

$$V_{xf} = 4.612\,\frac{m}{s}$$

$$t_{left} = \frac{V_{xf} - V_{xi}}{a_x}$$

$$t_{left} = \frac{0 - 4.612\,\frac{m}{s}}{-12.5\,\frac{m}{s^2}}$$

The time traveled to maximum horizontal displacement:

$$t_{left} = 3.69\cdot 10^{-1} s$$

Final horizontal velocity:

Knowing the total horizontal displacement from above, we can use this knowing it will come back the same distance to hit the cliff and we can now find the final horizontal velocity:

$$V_{xf}^2 = V_{xi}^2 + 2\cdot a_x\cdot\Delta x$$

$$V_{xf} = \sqrt{\left(4.612\,\frac{m}{s}\right) + 2\cdot\left(12.5\,\frac{m}{s^2}\right)(1.046\,m)}$$

The final horizontal velocity:

$$V_{xf} = 6.886\,\frac{m}{s}$$

Total time below the cliff:

We can now find for the remainder of the time below the cliff:

$$V_{xf} = V_{xi} + a_x\cdot t$$

$$t = \frac{V_{xf} - V_{xi}}{a_x}$$

$$t = \frac{6.886\,\frac{m}{s} - 4.612\,\frac{m}{s}}{12.5\,\frac{m}{s^2}}$$

$$t = 1.819\cdot 10^{-1} s$$

We now have the total time below the cliff:

$$t_{total\,below\,cliff} = t + 2\cdot t_{left}$$

$$t_{total\,below\,cliff} = 0.1819\,s + 2\cdot 0.369\,s$$

The final vertical displacement:

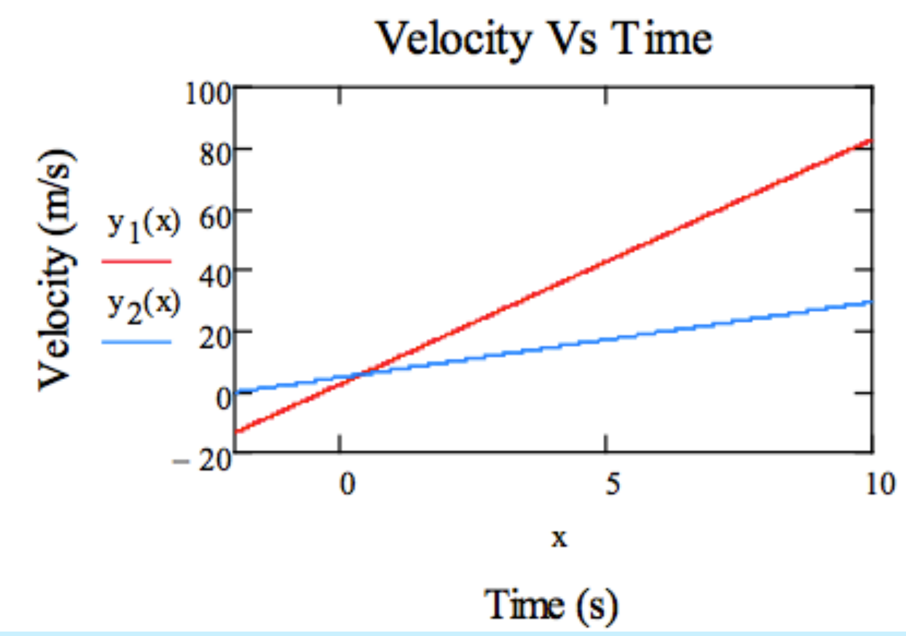
Finally we can use the time spent below the cliff to find the displacement in the vertical direction.

$$\Delta y = V_{yi}\cdot t + \frac{1}{2}a_y\cdot t^2$$

$$\Delta y = \left(1.53\,\frac{m}{s}\right)(0.9199\,s) + \frac{1}{2}\left(10.33\,\frac{m}{s^2}\right)(0.9199)^2$$

$$\Delta y = 5.778\,m$$

The ball will hit the cliff 5.778 meters below the ledge due to the magnetic field.



Conclusion:

This problem shows the use of kinematics equations which can be used to solve for displacement when given the initial position and velocity and when the acceleration is constant. In most physics problems the kinematics equations are used to solve problems dealing with projectile motion on earth. When this is the case, the vertical acceleration is –g (-g=-9.8m/s^2) and there is no horizontal acceleration. This problem generalizes the use of kinematics equations by using a different planet which has a different acceleration due to gravity and by using a planet with large magnetic fields which produce constant horizontal acceleration. The graph above gives an idea of how the velocity of the ball changed due to the magnetic field acting on it. The (red) velocity above the cliff was gaining speed at a much higher rate than the (blue) velocity below the cliff. Lastly, we showed that the cliff itself does not need to be infinitely tall for the ball to hit the cliff instead of the ground, but at most 6 meters tall.