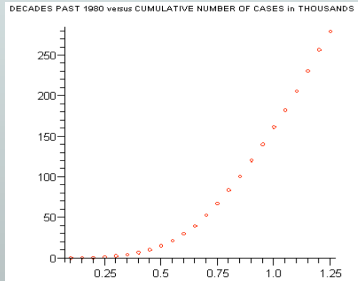
Linear Ideas Everywhere!!!!

Choose Ohio First

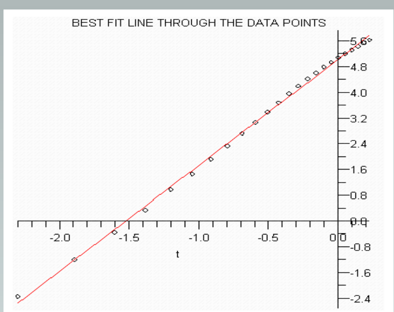
Anna Goodman, Advisor: Dr. Rodrigues

Linearization of Non-Linear Functions: modeling the increase in AIDS cases.

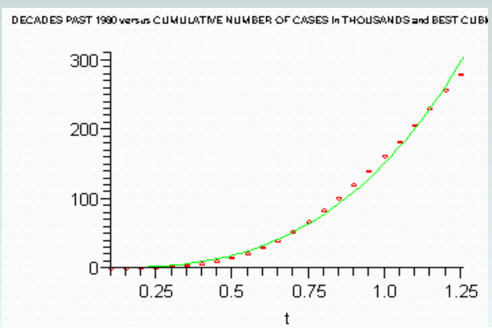
Let c be the sum of all reported cases since 1980 (in thousands) and t be the time in decades past 1980 or (year-1980)/10. Now we graph c vs. t.



The above graph shows that the data is not linear. We want to see if the relation can be expressed as a power rule. We will assume that $c=A t^k$ or equivalently log(c)=k*log(t)+log(A). When the points are plotted in a log-log scale we see a linear relation in the new variables: log(t) and log(c). The graph below also shows the line that best fits the points.

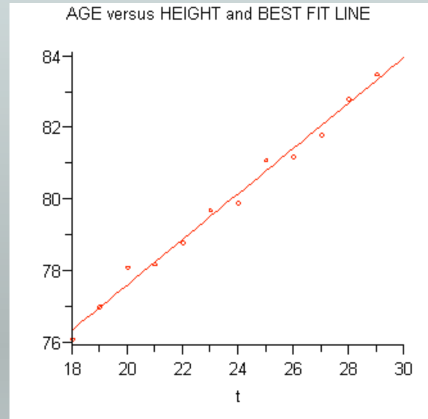


Rescaling back to the original variables allow us to calculate a best curve fit: $c=155^*t^3$



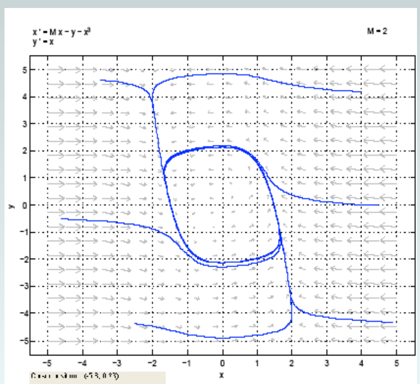
Linear Regression

Heights of 161 children ages 18-30 months were taken in the village of Kalama in Egypt. The average heights of the children are graphed below. We used linear regression and the method of least squares to find the best fit line given by height=0.65*age + 64.93. We can use this line to predict the height of a child age 18-36 months in this village. For example, a 27.5 child could be expected to be about 82.4 cm.



Van der Pol Oscillations

Van der Pol modeled a closed loop electrical circuits using a system of differential equations. The equations also model the displacement y and the velocity x of a mass-spring system with frictional force. To draw the oscillatory behavior we draw at each point a directed line segment (vector). For example, for the model below, at the point (1,-2), the line segment has slope 1/3. Each vectors give us the direction and intensity of the flow (or mass). The electrical circuit (or spring-mass system) below has a limit cycle.



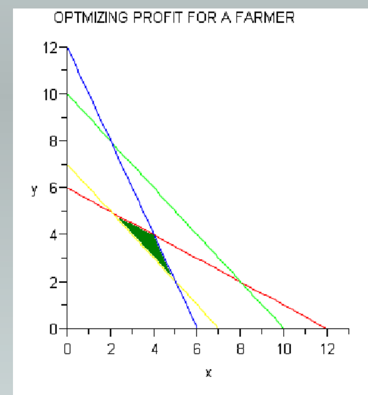
Linear Programming

In linear programming we are given a set of constraints. We graph these inequalities to find the region surrounded by the inequalities. We know that the maximum and minimum values are at the corners of this section.

Here's an example: A farmer has 10 acres to plant in wheat and rye. He has to plant at least 7 acres. However, he has only $1200 to spend and each acre of wheat costs $200 to plant and each acre of rye costs $100 to plant. Moreover, the farmer has to get the planting done in 12 hours and it takes an hour to plant an acre of wheat and 2 hours to plant an acre of rye. If the profit is $500 per acre of wheat and $300 per acre of rye how many acres of each should be planted to maximize profits?

We will let x equal the acres of wheat and y the acres of rye. We want to maximize profit. The profit is 500x+300y. Our constraints are:

$10 \geq x+y > 7$, $200x+100y \leq 1200$, $12 \geq x+2y$. The graph below shows these constraints.



The shaded triangle with corners (5,2), (4,4) and (2,5). is the region of interest. We test these points and find that the profit is maximized when we plant 4 of each crop which gives us a $3200 profit.

References

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